Glossary
COMP6741: Parameterized and Exact Computation

19T3

Glossary

acyclic: A graph is acyclic if it has no cycle as a subgraph.

bipartite: A graph $G = (V, E)$ is bipartite if its vertex set can be partitioned into two independent sets. A partition (A, B) of V into independent sets is called a bipartition of G. The graph G is then often denoted by $G = (A \uplus B, E)$.

Boolean formula: A Boolean formula is constructed from Boolean variables that can take the values true and false (or 1 and 0) by the following operations: conjunction (AND, $\wedge$), disjunction (OR, $\vee$), and negation (NOT, $\neg$).

clique: A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a clique in G if $G[S]$ is a complete graph.

closed neighborhood: The closed neighborhood of a vertex v in a graph G is $N_G[v] := \{v\} \cup N_G(v)$. The subscript may be omitted if G is clear from the context.

closed set neighborhood: The closed neighborhood of a subset of vertices $S \subseteq V$ in a graph $G = (V, E)$ is $N_G[S] := \bigcup_{v \in S} N_G[v]$. The subscript may be omitted if G is clear from the context.

coloring: A coloring of a graph $G = (V, E)$ is a function from V to a set of colors (integers) such that every two adjacent vertices in G are mapped to different colors. A k-coloring is a coloring using exactly k colors.

complete: A graph G is complete if there is an edge between each pair of vertices in G. A complete graph on n vertices is denoted by K_n.

Conjunctive Normal Form: A Boolean formula is in Conjunctive Normal Form if it is a conjunction of clauses, each clause is a disjunctions of literals, and each literal is a Boolean variable or its negation..

connected: A graph G is connected if there is a walk between every two vertices of G.

connected component: Maximal connected subgraph.

cycle: 2-regular connected graph. A cycle on n vertices is denoted C_n.

degree: The degree of a vertex v in a graph G is $d_G(v) := |N_G(v)|$. The subscript may be omitted if G is clear from the context. The degree of a vertex v in a **multigraph:** G is the number of times v appears as an end point of an edge in E..

directed acyclic graph: A directed acyclic graph (DAG) is a directed graph that contains no directed cycle as a directed subgraph.

directed cycle: Orientation of a cycle where each vertex has in-degree 1.

directed graph: A directed graph G is an ordered pair (V, A) of a set V of vertices and a set A of arcs, where A is a set of ordered pairs of vertices. Its vertex set is $V(G) = V$ and its arc set is $A(G) = A$.

directed path: Orientation of a path where each vertex has in-degree 1, except the start vertex, which has in-degree 0 and out-degree 1.

disjoint union: For two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ with $V_1 \cap V_2 = \emptyset$, the disjoint union of G_1 and G_2, denoted $G_1 \oplus G_2$ is the graph $(V_1 \cup V_2, E_1 \cup E_2)$; in case $V_1 \cap V_2 \neq \emptyset$, vertices need to be renamed before we can take the disjoint union of these graphs.

distance: In a graph G, the distance between two vertices $u \in V$ and $v \in V$ is the length of the shortest walk minus one between u and v, that is the minimum number of edges needed to be traversed to reach v from u and it is denoted by $dist_G(u, v)$.

dominating set: A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a dominating set of G if $N_G[S] = V$.

feedback vertex set: A feedback vertex set of a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that $G - S$ is acyclic.

forest: Acyclic graph.

graph: A (simple, undirected) graph G is an ordered pair (V, E) of a set V of vertices and a set E of edges, where E is a set of unordered pairs of distinct vertices. Its vertex set is $V(G) = V$ and its edge set is $E(G) = E$.

in-degree: The in-degree of a vertex v in a directed graph D is $d^-_D(v) := |\{uv \in A\}|$. The subscript may be omitted if D is clear from the context.

independent set: A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is an independent set of G if $G[S]$ has no edges.

induced subgraph: For a graph $G = (V, E)$ and a vertex set $S \subseteq V$, the subgraph of G induced on S is the graph $G[S] := (S, \{uv \in E : u, v \in S\})$.

maximal (set): For a set $\mathcal{S}$ of subsets of a ground set U, a set $X \in \mathcal{S}$ is maximal if there exists no set $Y \in \mathcal{S}$ with $X \subsetneq Y$.

maximum (set): For a set $\mathcal{S}$ of subsets of a ground set U, a set $X \in \mathcal{S}$ is maximum if there exists no set $Y \in \mathcal{S}$ with $|Y| > |X|$.

maximum degree: The maximum degree of a graph $G = (V, E)$ is $\Delta(G) := \max_{v \in V} d_G(v)$.

minimum degree: The minimum degree of a graph $G = (V, E)$ is $\delta(G) := \min_{v \in V} d_G(v)$.

multigraph: A multigraph G is an ordered pair (V, E) of a set V of vertices and a *multi*set E of edges, where E is a multiset of unordered pairs of vertices. Its vertex set is $V(G) = V$ and its edge set is $E(G) = E$.

open neighborhood: The (open) neighborhood of a vertex v in a graph $G = (V, E)$ is $N_G(v) := \{u \in V : uv \in E\}$. The subscript may be omitted if G is clear from the context.

open set neighborhood: The (open) neighborhood of a subset of vertices $S \subseteq V$ in a graph $G = (V, E)$ is $N_G(S) := N_G[S] \setminus S$. The subscript may be omitted if G is clear from the context.

order of growth: Let $g : \mathbb{R}_+ \to \mathbb{R}_+$ be a function. The set $O(g(n))$ contains every function f such that there exist $c, n_0 \geq 0$ such that $f(n) \leq c \cdot g(n)$ for every $n \geq n_0$. The set $o(g(n))$ contains every function f such that for every $\epsilon > 0$ there exists a $n_0 \geq 0$ such that $f(n) \leq \epsilon \cdot g(n)$ for every $n \geq n_0$. For the set $\Omega(g(n))$, we have that $f(n) \in \Omega(g(n))$ iff $g(n) \in O(f(n))$. For the set $\omega(g(n))$, we have that $f(n) \in \omega(g(n))$ iff $g(n) \in o(f(n))$. For the set $\Theta(g(n))$, we have that $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.

orientation: An orientation of a graph G is a directed graph D that has exactly one arc for each edge of G with the same endpoints.

out-degree: The out-degree of a vertex v in a directed graph D is $d^+_D(v) := |\{vu \in A\}|$. The subscript may be omitted if D is clear from the context.

path: Tree with maximum degree at most 2. A path on n vertices is denoted P_n.

regular: A graph is d-regular if each of its vertices has degree d. A graph is regular if it is d-regular for some d.

subgraph: A graph H is a subgraph of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

tree: Acyclic, connected graph.

vertex cover: A subset of vertices $S \subseteq V$ of a graph $G = (V, E)$ is a vertex cover of G if each edge of G is incident to at least one vertex of S.

vertex removal: For a graph $G = (V, E)$ and a vertex set $S \subseteq V$, the graph obtained by removing S from G is $G - S := G[V \setminus S]$. If $S = \{u\}$, we may write $G - u$ instead of $G - \{u\}$.

walk: Sequence of vertices in a graph, with each vertex being adjacent to the vertices immediately preceding and succeeding it in the sequence.

Problem Definitions

k-Coloring

Given a graph G, determine if there is a coloring of G with at most k colors.

k-Sat

Given a Boolean formula in Conjunctive Normal Form where each clause has at most k literals, determine if there is an assignment of its variables such that the formula evaluates to `true`.

Dominating Set

Given a graph G and an integer k, determine whether G has a dominating set of size k.

Feedback Vertex Set

Given a (multi)graph G and an integer k, determine whether G has a feedback vertex set of size at most k.

Independent Set

Given a graph G and an integer k, determine whether G has an independent set of size k.

Maximum Independent Set

Given a graph G, find an independent set of G of maximum cardinality.

Minimum Vertex Cover

Given a graph G, find a vertex cover of G of minimum cardinality.

Sat

Given a Boolean formula, determine if there is an assignment of its variables such that the formula evaluates to true.

Traveling Salesman Problem

Given a set $\{1, \ldots, n\}$ of n cities, the distance $d(i, j)$ between every two cities i and j, and an integer k, determine whether there is a tour with total distance at most k. A *tour* is a permutation of the cities starting and ending in city 1.

Vertex Cover

Given a graph G and an integer k, determine whether G has a vertex cover of size k.