Exercise sheet 3

COMP6741: Parameterized and Exact Computation

Serge Gaspers

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Exercise 1. Suppose there exists a $O^*(1.2^n)$ time algorithm, which, given a graph G on n vertices, computes **the size** of a largest independent set of G.

Design an algorithm, which, given a graph G, **finds** a largest independent set of G in time $O^*(1.2^n)$.

Exercise 2. Let A be a branching algorithm, such that, on any input of size at most n its search tree has height at most n and for the number of leaves $L(n)$, we have

$$L(n) = 3 \cdot L(n-2)$$

Upper bound the running time of A, assuming it spends only polynomial time at each node of the search tree.

Exercise 3. Same question, except that

$$L(n) \leq \max \begin{cases} 2 \cdot L(n-3) \\ L(n-2) + L(n-4) \\ 2 \cdot L(n-2) \\ L(n-1) \end{cases}$$

Exercise 4. Consider the Max 2-CSP problem

Max 2-CSP	
Input:	A graph $G = (V, E)$ and a set S of *score functions* containing • a score function $s_e : \{0,1\}^2 \to \mathbb{N}_0$ for each edge $e \in E$, • a score function $s_v : \{0,1\} \to \mathbb{N}_0$ for each vertex $v \in V$, and • a score "function" $s_\emptyset : \{0,1\}^0 \to \mathbb{N}_0$ (which takes no arguments and is just a constant convenient for bookkeeping).
Output:	The maximum score $s(\phi)$ of an assignment $\phi : V \to \{0,1\}$: $$s(\phi) := s_\emptyset + \sum_{v \in V} s_v(\phi(v)) + \sum_{uv \in E} s_{uv}(\phi(u), \phi(v)).$$

1. Design simplification rules for vertices of degree ≤ 2.
2. Using the simple analysis, design and analyze an $O^*(2^{m/4})$ time algorithm, where $m = |E|$.
3. Use the measure $\mu := w_e \cdot m + \left(\sum_{v \in V} w_{d_G(v)}\right)$ to improve the analysis to $O^*(2^{m/5})$.