Lab 2

COMP9021, Session 2, 2015

1 ☞ Circular jail cell

Adapted from `https://www.ocf.berkeley.edu/~wwu/riddles/hard.shtml`

A circular jail has 100 cells numbered from 1 to 100. Each cell has an inmate and the door is locked. One night the jailor gets drunk and starts running around the jail in circles. In his first round he opens each door. In his second round he visits every 2nd door (2, 4 , 6, ...) and shuts the door. In the 3rd round he visits every 3rd door (3, 6, 9, ...) and if the door is shut he opens it, if it is open he shuts it. This continues for 100 rounds and exhausted the jailor falls down. Write a python program that outputs how many prisoners found their doors open after 100 rounds.

2 ☞ Perfect numbers

A number is perfect if it is equal to the sum of its divisors, itself excluded. For instance, the divisors of 28 distinct from 28 are 1, 2, 4, 7 and 14, and $1 + 2 + 4 + 7 + 14 = 28$, hence 28 is perfect.

Write a program that outputs all 3-digit perfect numbers.

You should find out that there is only one solution.

3 ☞ Finding special triples of the form $(n, n+1, n+2)$

Write a program that finds all triples of consecutive positive three-digit integers each of which is the sum of two squares, that is, all triples of the form $(n, n+1, n+2)$ such that:

- n, $n+1$ and $n+2$ are integers at least equal to 100 and at most equal to 999;
- each of n, $n+1$ and $n+2$ is of the form $a^2 + b^2$.

Hint: As we are not constrained by memory space for this problem, we might use a list that stores an integer for all indexes n in $[100, 999]$, equal to 1 in case n is the sum of two squares, and to 0 otherwise. Then it is just a matter of finding three consecutive 1's in the list. This idea can be refined (by not storing 1s, but suitable nonzero values) to not only know that some number is of the form $a^2 + b^2$, but also know such a pair (a, b)...

The output of the program could be:

```
(144, 145, 146) (equal to (0^2+12^2, 8^2+9^2, 5^2+11^2)) is a solution.
(232, 233, 234) (equal to (6^2+14^2, 8^2+13^2, 3^2+15^2)) is a solution.
(288, 289, 290) (equal to (12^2+12^2, 8^2+15^2, 11^2+13^2)) is a solution.
(360, 361, 362) (equal to (6^2+18^2, 0^2+19^2, 1^2+19^2)) is a solution.
(520, 521, 522) (equal to (14^2+18^2, 11^2+20^2, 9^2+21^2)) is a solution.
(576, 577, 578) (equal to (0^2+24^2, 1^2+24^2, 17^2+17^2)) is a solution.
(584, 585, 586) (equal to (10^2+22^2, 12^2+21^2, 15^2+19^2)) is a solution.
(800, 801, 802) (equal to (20^2+20^2, 15^2+24^2, 19^2+21^2)) is a solution.
(808, 809, 810) (equal to (18^2+22^2, 5^2+28^2, 9^2+27^2)) is a solution.
```

(The decompositions into sums of squares could differ.)

4 Estimating the probabilities of hypotheses in the light of more and more evidence

Write a program that simulates the cast of an unknown die, chosen from a set of 5 dice with 4, 6, 8, 12, and 20 faces. To start with, every die has a probability of 0.2 to be the chosen die. At every cast, the probability of each die being the chosen die is updated using Bayes' rule (find out about it if you do not know it, it is simple...). The probabilities are displayed for at most 5 casts. If more than 5 casts have been requested, the final probabilities obtained for the chosen number of casts are eventually displayed.

Here is a possible output:

```
Enter the desired number of times a randomly chosen die will be cast: 2

This is a secret, but the chosen die is the one with 4 faces

Casting the chosen die... Outcome: 4
The updated dice probabilities are:
 4: 37.04%
 6: 24.69%
 8: 18.52%
12: 12.35%
20: 7.41%

Casting the chosen die... Outcome: 1
The updated dice probabilities are:
 4: 54.18%
 6: 24.08%
 8: 13.55%
12: 6.02%
20: 2.17%
```

Here is another possible output:

```
Enter the desired number of times a randomly chosen die will be cast: 5

This is a secret, but the chosen die is the one with 8 faces

Casting the chosen die... Outcome: 7
The updated dice probabilities are:
 4: 0.00%
 6: 0.00%
 8: 48.39%
12: 32.26%
20: 19.35%

Casting the chosen die... Outcome: 1
The updated dice probabilities are:
 4: 0.00%
 6: 0.00%
 8: 62.33%
12: 27.70%
20: 9.97%

Casting the chosen die... Outcome: 1
The updated dice probabilities are:
 4: 0.00%
 6: 0.00%
 8: 73.51%
12: 21.78%
20: 4.70%

Casting the chosen die... Outcome: 3
The updated dice probabilities are:
 4: 0.00%
 6: 0.00%
 8: 81.76%
12: 16.15%
20: 2.09%

Casting the chosen die... Outcome: 7
The updated dice probabilities are:
 4: 0.00%
 6: 0.00%
 8: 87.57%
12: 11.53%
20: 0.90%
```

Here is still another possible output:

```
Enter the desired number of times a randomly chosen die will be cast: 20

This is a secret, but the chosen die is the one with 6 faces

Casting the chosen die... Outcome: 1
The updated dice probabilities are:
 4: 37.04%
 6: 24.69%
 8: 18.52%
12: 12.35%
20: 7.41%

Casting the chosen die... Outcome: 4
The updated dice probabilities are:
 4: 54.18%
 6: 24.08%
 8: 13.55%
12: 6.02%
20: 2.17%

Casting the chosen die... Outcome: 6
The updated dice probabilities are:
 4: 0.00%
 6: 63.54%
 8: 26.80%
12: 7.94%
20: 1.72%

Casting the chosen die... Outcome: 6
The updated dice probabilities are:
 4: 0.00%
 6: 72.10%
 8: 22.81%
12: 4.51%
20: 0.58%

Casting the chosen die... Outcome: 5
The updated dice probabilities are:
 4: 0.00%
 6: 78.68%
 8: 18.67%
12: 2.46%
20: 0.19%

The final probabilities are:
 4: 0.00%
 6: 99.68%
 8: 0.32%
12: 0.00%
20: 0.00%
```