Searching and Trees

- COMP1927 Computing 17x1
- Sedgewick Chapters 5, 12

Searching (cont)

Searching is a very important/frequent operation. Several approaches have been developed:

- *O(n)* ... linear scan (search technique of last resort)
- *O(logn)* ... binary search, **search trees** (trees also have other uses)
- *O(1)* ... **hash tables** (only *O(1)* under optimal conditions)

Searching (cont)

Linear structures: arrays, linked lists

Arrays = random access.

Lists = sequential access.

	Array	**List**
Unsorted	$O(n)$ *(linear scan)*	$O(n)$ *(linear scan)*
Sorted	$O(\log n)$ *(binary search)*	$O(n)$ *(linear scan)*

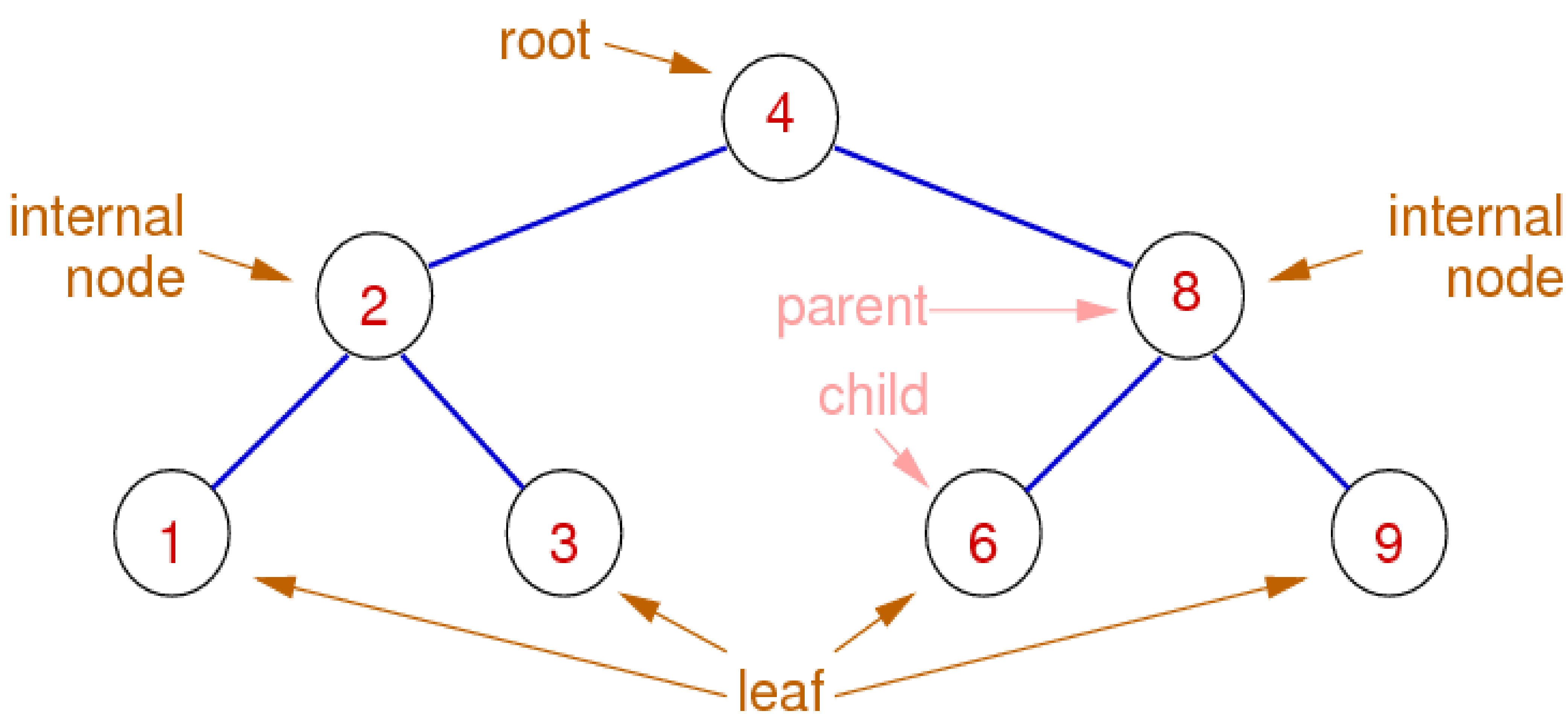
SEARCHING

- Storing and searching sorted data:
- Dilemma: Inserting into a sorted sequence
  - Finding the insertion point on an array – O(log n) but then we have to move everything along to create room for the new item
  - Finding insertion point on a linked list O(n) but then we can add the item in constant time.
- Can we get the best of both worlds?

Tree terminology

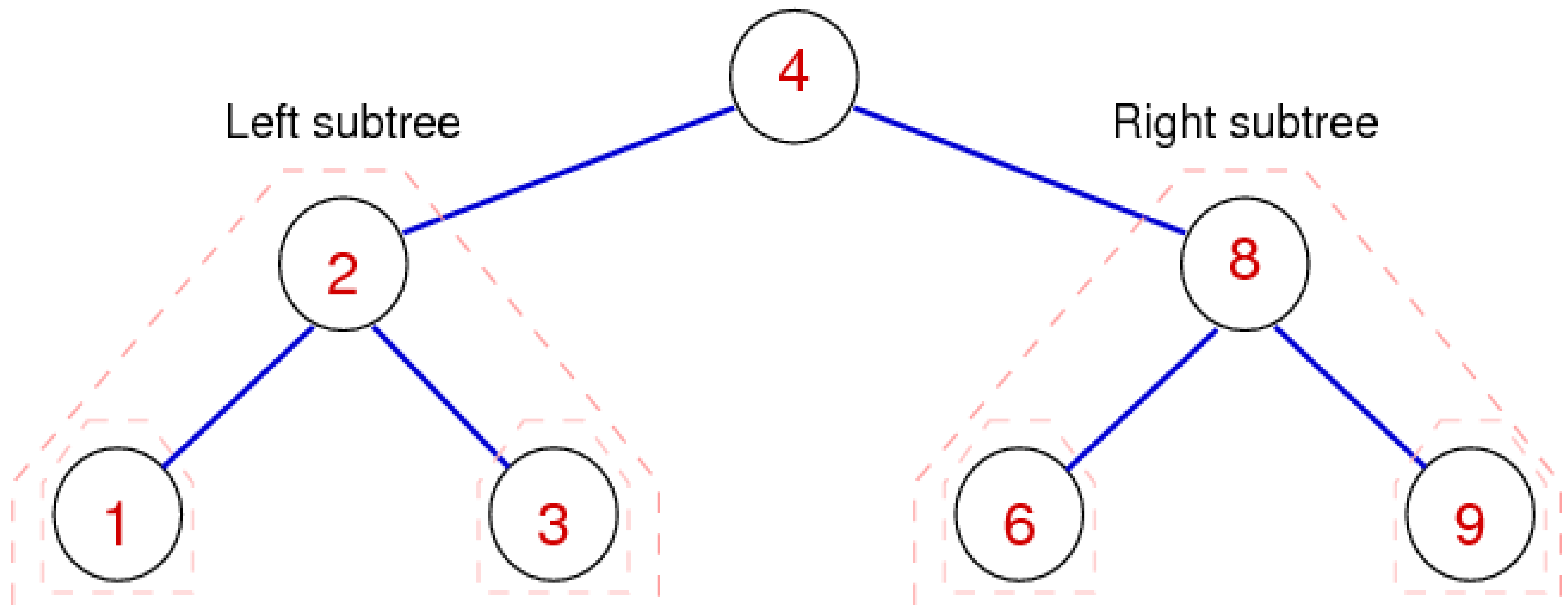
Trees are branched data structures

- consisting of *nodes (vertices)* and *links (edges)*, with no cycles
- each node contains a *data* value (or key + data)
- each node has links to $\leq k$ other nodes (k=*2* below)



Trees

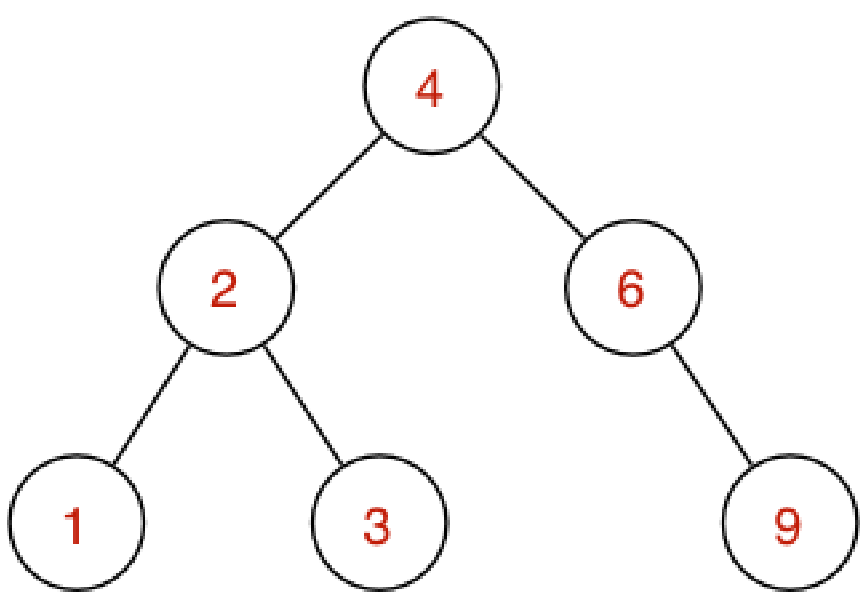
- Trees can be viewed as a set of nested structures: each node has k possibly empty subtrees



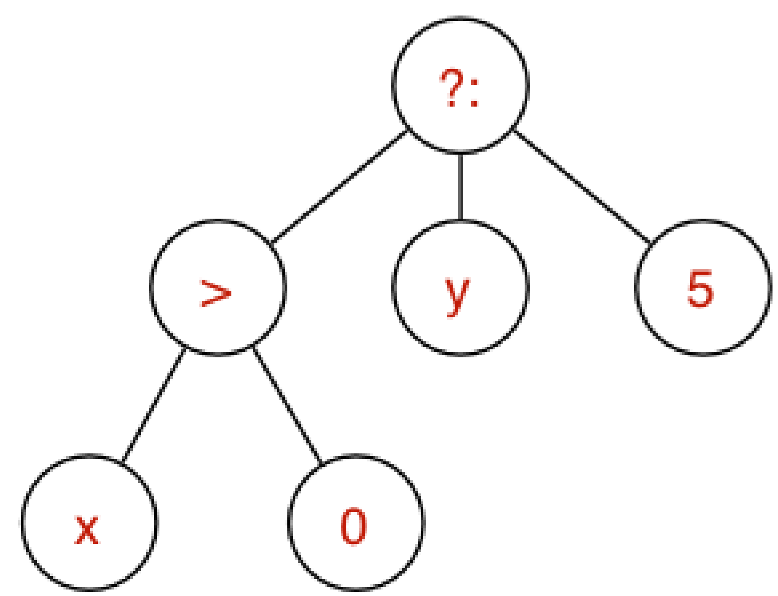
USES OF TREES

- Trees are used in many contexts, e.g. representing hierarchical data structures (e.g. expressions)
- efficient searching (e.g. sets, symbol tables, ...)

Search Tree

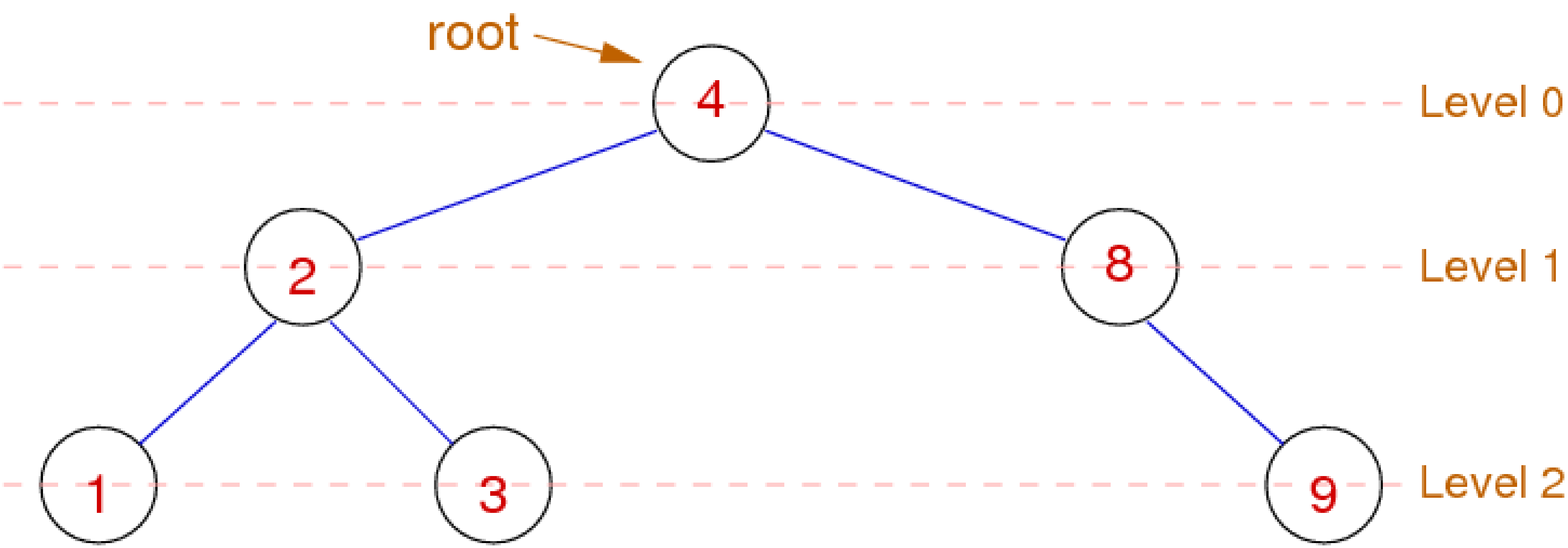


Expression Tree



Tree Terminolgy

- Level of a node in a tree (or depth) is one higher than the level of its parent
  - Depth of the root is 0
- We call the length of the longest path from the root to a node the height of a tree



Special properties of some trees

- M-ary tree: each internal node has exactly M children
- Ordered tree: order of the children at every node is specified through constraints on the data/keys in the nodes
- Balanced tree: a tree with properties that
  - #nodes in left subtree = #nodes in right subtree
  - this property applies over all nodes in the tree

Binary trees

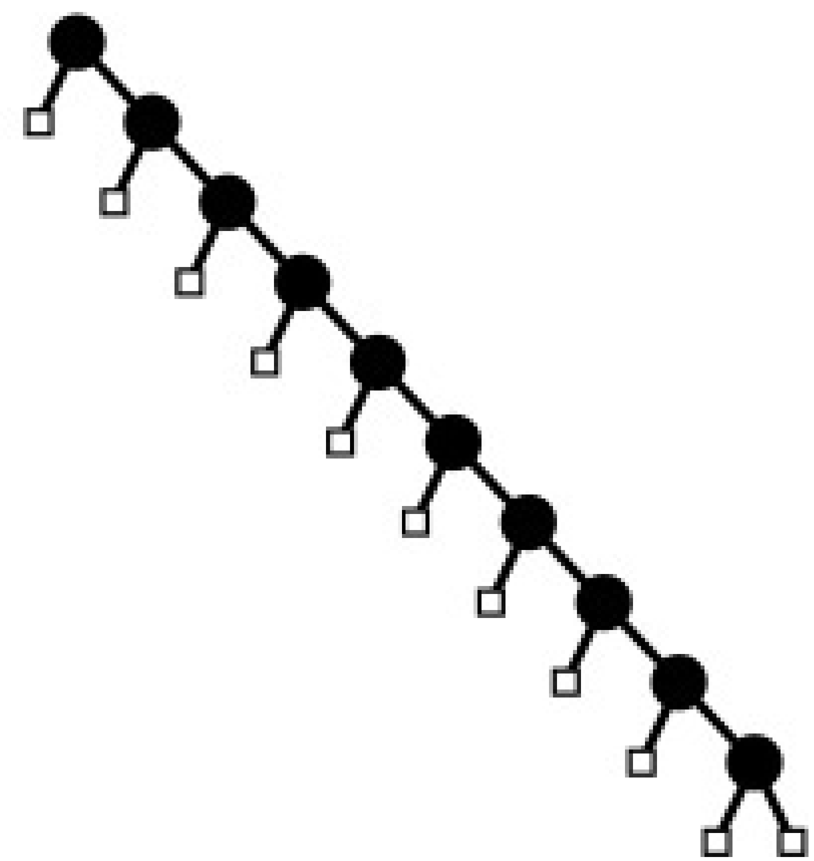
For much of this course, we focus on *binary trees*

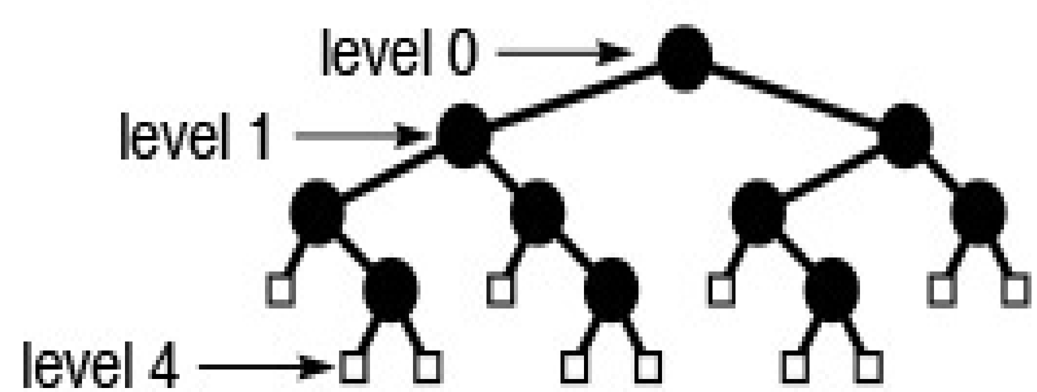
A *binary tree* (simplest type of M-ary tree) is defined recursively, as being either:

- empty (contains no nodes)
- consisting of a node, with two sub-trees
  - each node contains a value
  - the left and right sub-trees are *binary trees*

BINARY TREES: PROPERTIES

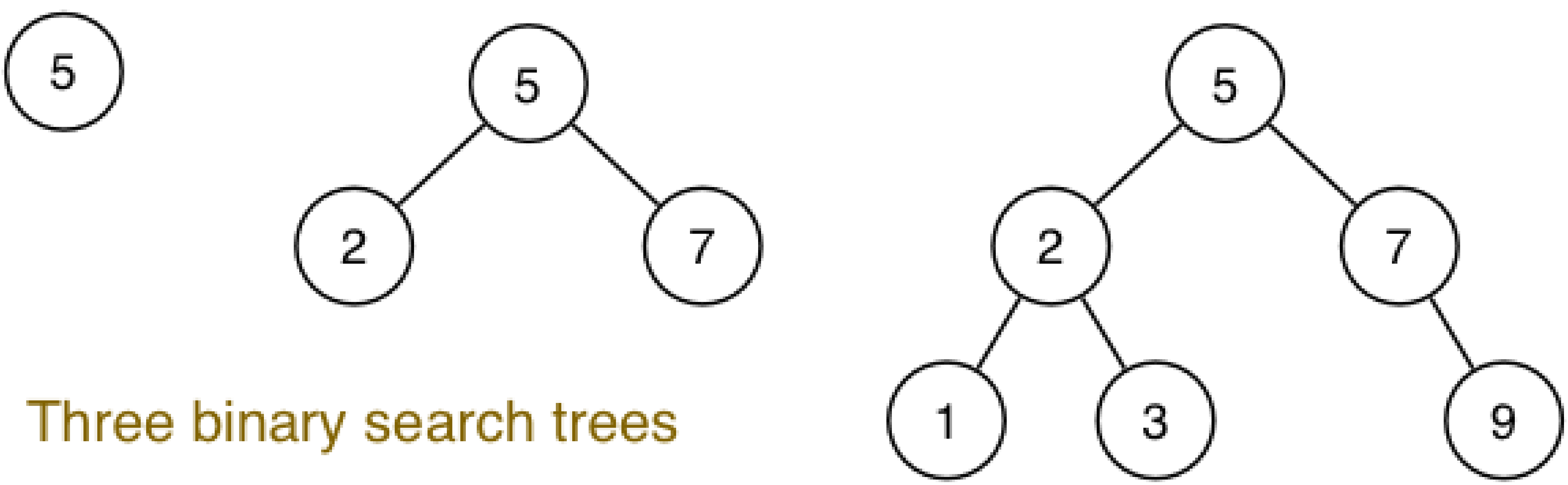
- A binary tree with *n* nodes has a height of
  - at most
    - n-1 (if degenerate) (an unbalanced tree, where for each parent node, there is only one child node)
  - at least
    - floor($\log_2$(n)) (if balanced)





BINARY SEARCH TREE (BST)

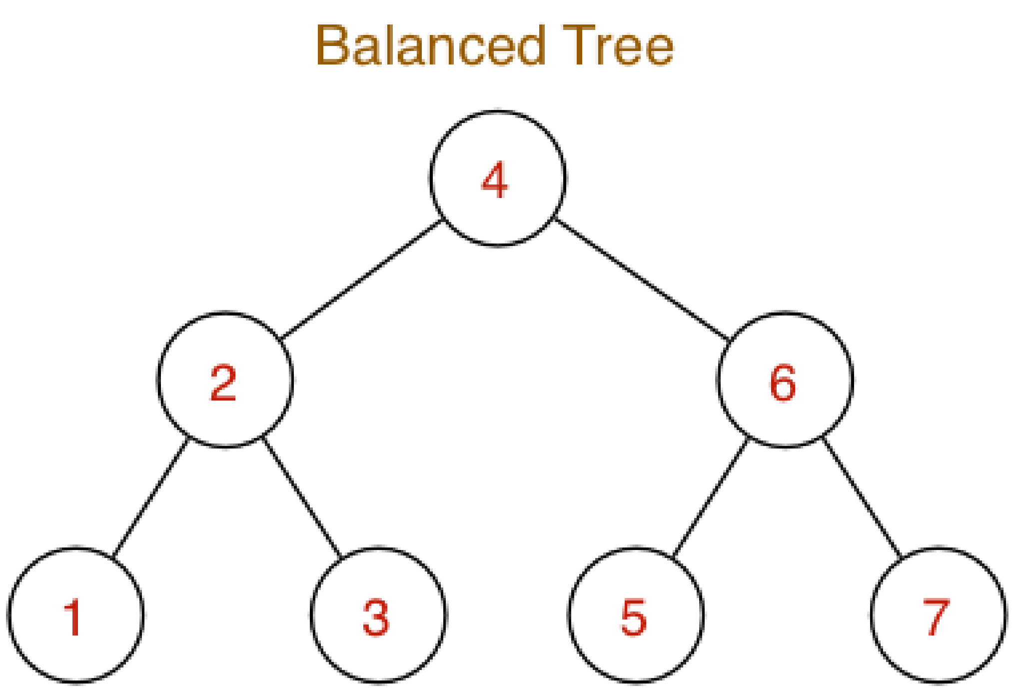
- A BST is an **ordered binary tree** that has:
  - all values in left sub-tree being less than root
  - all values in right sub-tree are greater than root
  - this property applies over all nodes in the tree
  - each node is the root of 0, 1 or 2 sub-trees

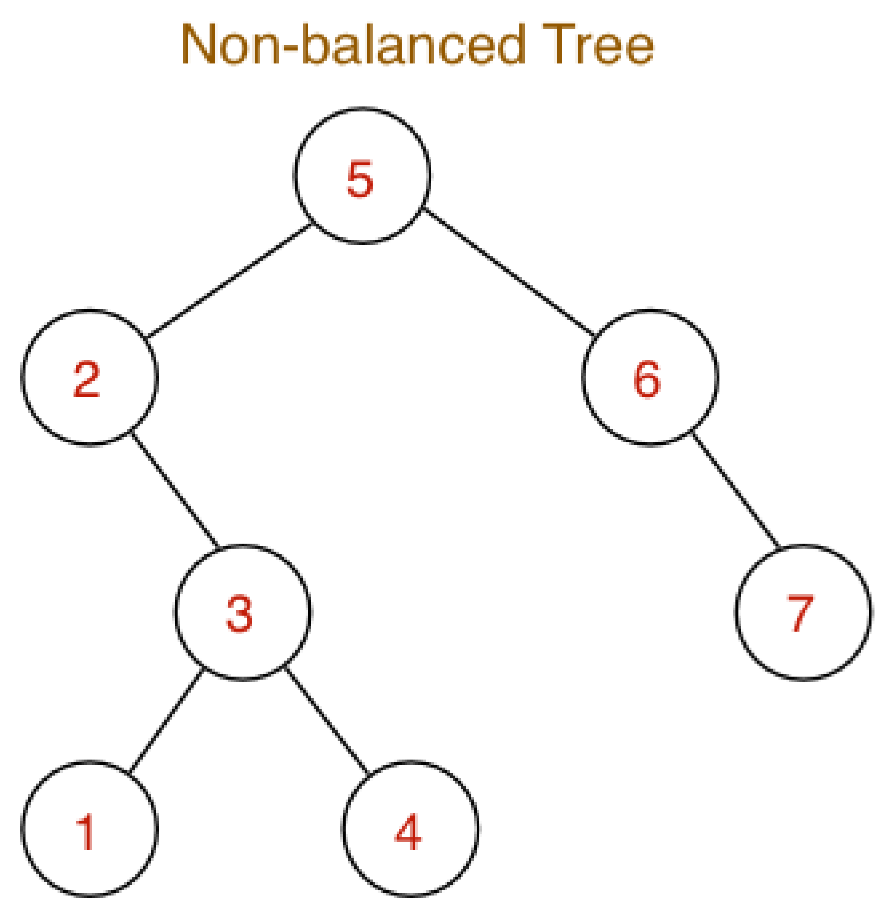


Three binary search trees

Binary Trees

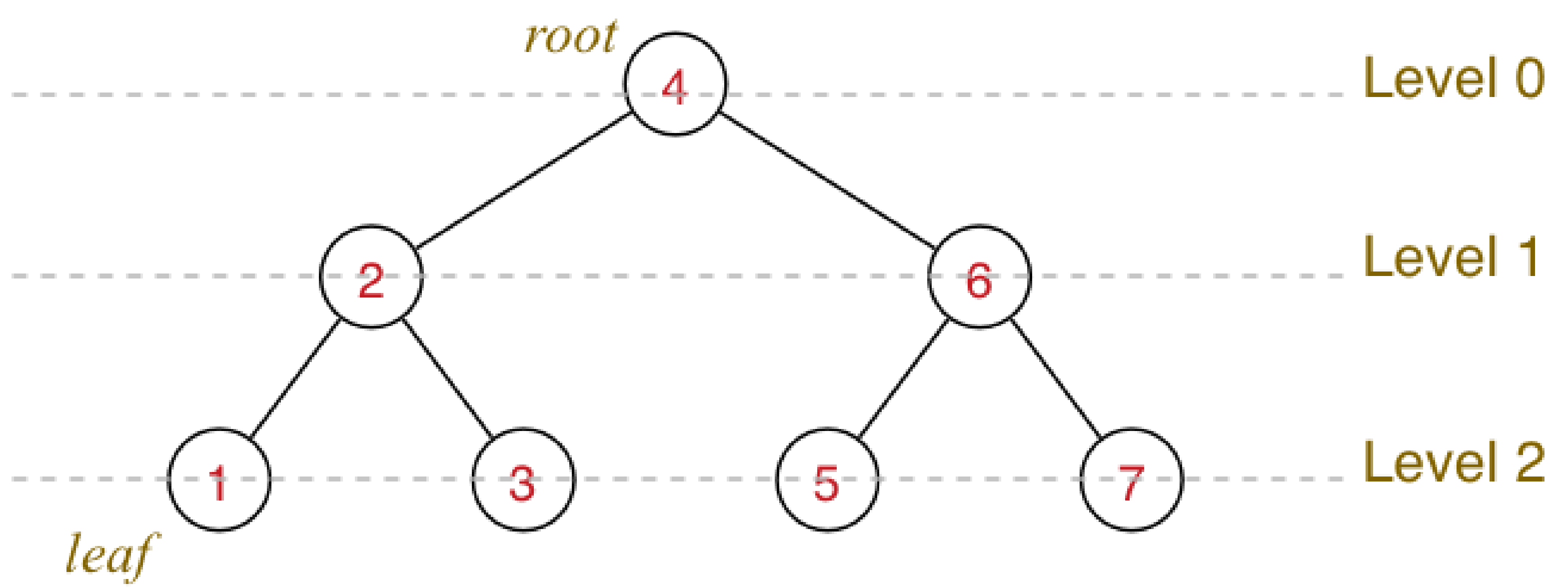
Shape of tree is determined by the order of insertion





Binary Search Trees

Depth of tree = max path length from root to leaf



Depth of tree with n nodes: min = $floor(log_2 n)$, max = n-1

Height balanced tree: ∀ nodes, depth(left subtree) ≅ depth(right subtree)

Time complexity of tree algorithms is typically *O(depth)*

Exercise: Insertion into BSTs

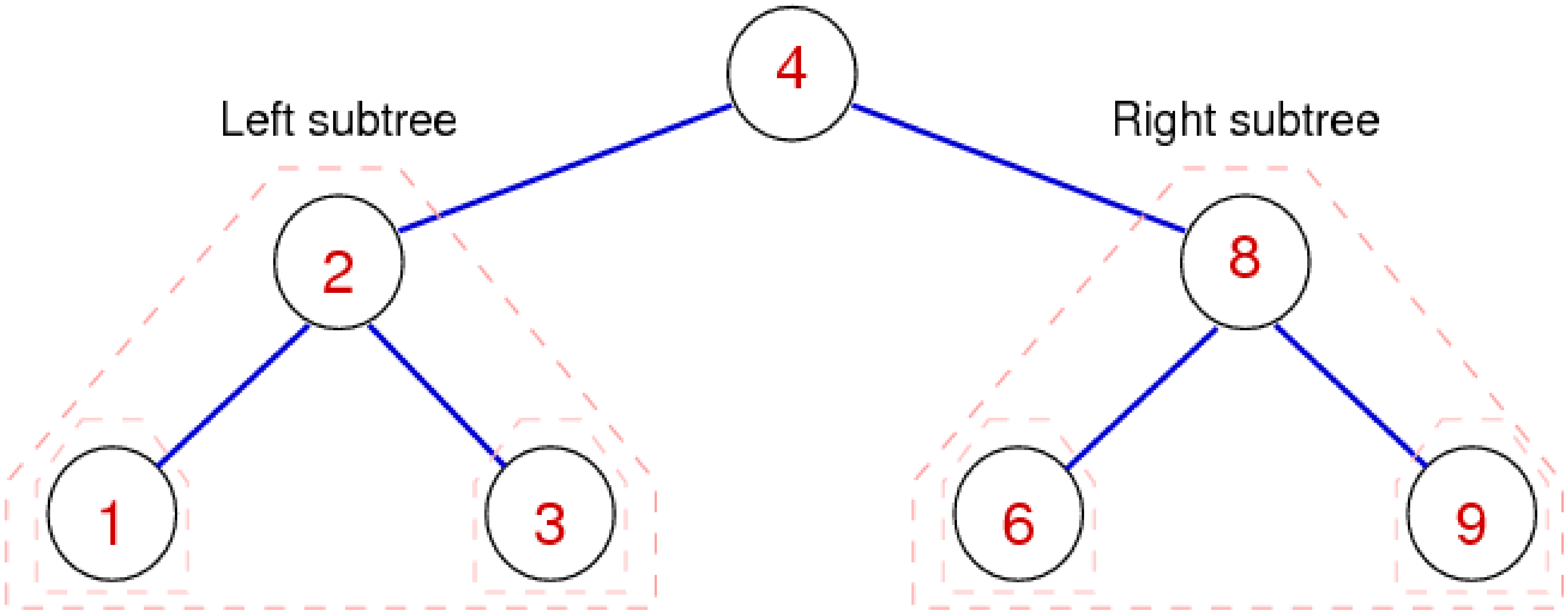
- For each of the sequences below start from an initially empty binary search tree
  - show the tree resulting from inserting the values in the order given
  - What is the height of each tree?
- (a) 4 2 6 5 1 7 3
- (b) 5 3 6 2 4 7 1
- (c) 1 2 3 4 5 6 7

Trees: Traversal

- For trees, several well-defined visiting orders exist:
  - Depth first traversals
    - preorder (NLR) … visit root, then left subtree, then right subtree
    - inorder (LNR) … visit left subtree, then root, then right subtree
    - postorder (LRN) … visit left subtree, then right subtree, then root
  - Breadth-first traversal or level-order … visit root, then all its children, then all their children

Example of traversals on a Binary tree

- Pre-Order: 4 2 1 3 8 6 9
- In-Order: 1 2 3 4 6 8 9
- Post-Order 1 3 2 6 9 8 4
- Level-Order: 4 2 8 1 3 6 9



Representing BSTs

A binary search tree is a generalization of a linked list:

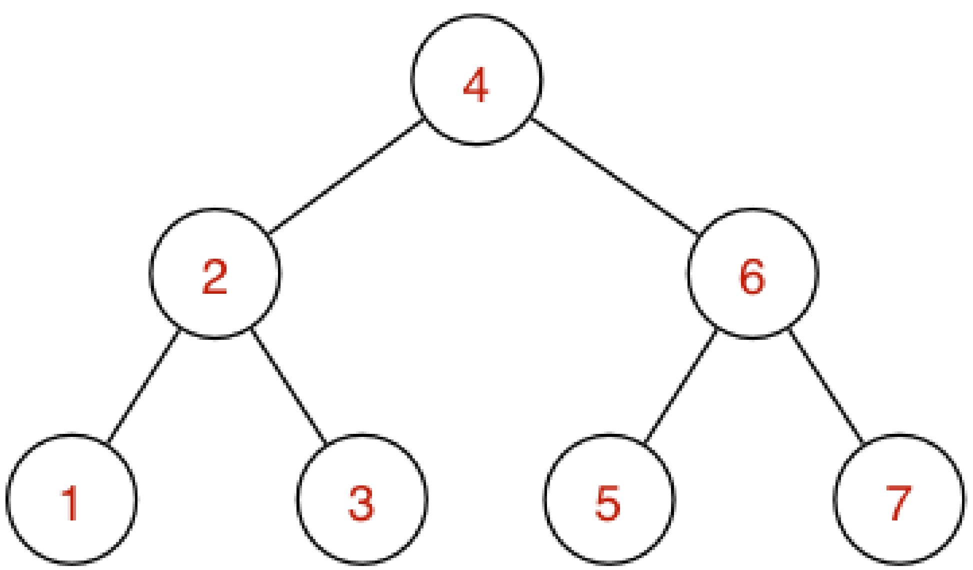
- nodes are a structure with two links to nodes
- empty trees are NULL links

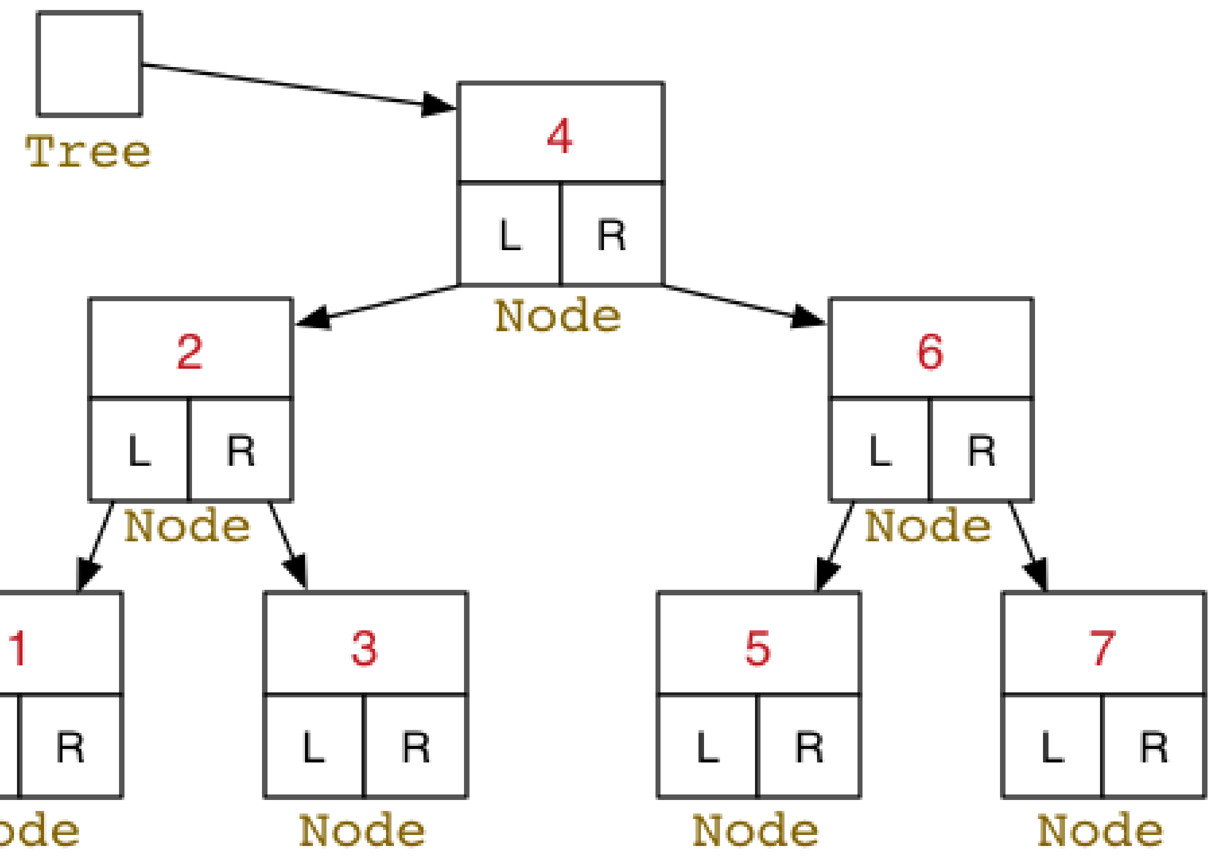
```
typedef struct treenode *Treelink;

struct treenode {
   int data;
   Treelink left, right;
}
```

Reprsenting BSTs

Abstract data vs concrete data





Binary Search Trees

Operations on BSTs:

- traverse(TreeLink, (*visit)) … traverse tree
- insert(TreeLink, Item) ... add new item to tree via key
- delete(TreeLink, Item) ... remove item with specified key from tree
- search(TreeLink,Item) ... find item containing key in tree
- height(TreeLink) … compute depth of tree
- nodes(TreeLink) … count #nodes in tree
- plus, "bookeeping" ... new(), dispose(), show(), empty(), ...

Notes: keys are unique (not technically necessary)

Binary Search Trees

Traversal (with parameterised visit option)

```
void traverse(TreeLink t, void (*visit)(Item)) {

    if (t == null) return;
    (*visit)(t);   // NLR traversal
     traverse(t->left, visit);
     // put "visit data" here for LNR
     traverse(t->right, visit);
     // put "visit data" here for LRN
}
```

Searching in BSTs

- **Recursive version**

```
// Returns non-zero if item is found,
// zero otherwise
int search(TreeLink  n, Item i){
    int result;
    if(n == NULL){
      result =  0;
    }else if(i < n->data){
       result = search(n->left,i);
    }else if(i > n->data)
       result = search(n->right,i);
    }else{ // you found the item
      result = 1;
    }
    return result;
}
```

* Exercise: Try writing an iterative version

Insertion into a BST

- Cases for inserting value V into tree T:
  - T is empty, make new node with V as root of new tree
  - root node contains V, tree unchanged (no dups)
  - V < value in root, insert into left subtree (recursive)
  - V > value in root, insert into right subtree (recursive)
- Non-recursive insertion of V into tree T:
  - search to location where V belongs, keeping parent
  - make new node and attach to parent
  - whether to attach L or R depends on last move

Insertion into a BST

```
//Returns the root of the tree
//Inserts duplicates on theleft hand side of tree
Treelink insertRec (Treelink tree, TreeItem item) {
    if(tree == NULL){
        Treelink newNode = createNode(item);
        return newNode; //now the root of the tree
    } else {
        if(item <= tree->item){
            tree->left = insertRec(tree->left, item); //
        } else {
            tree->right = insertRec(tree->right, item);
        }
    }
    return tree;
}
```

Deletion from BSTs

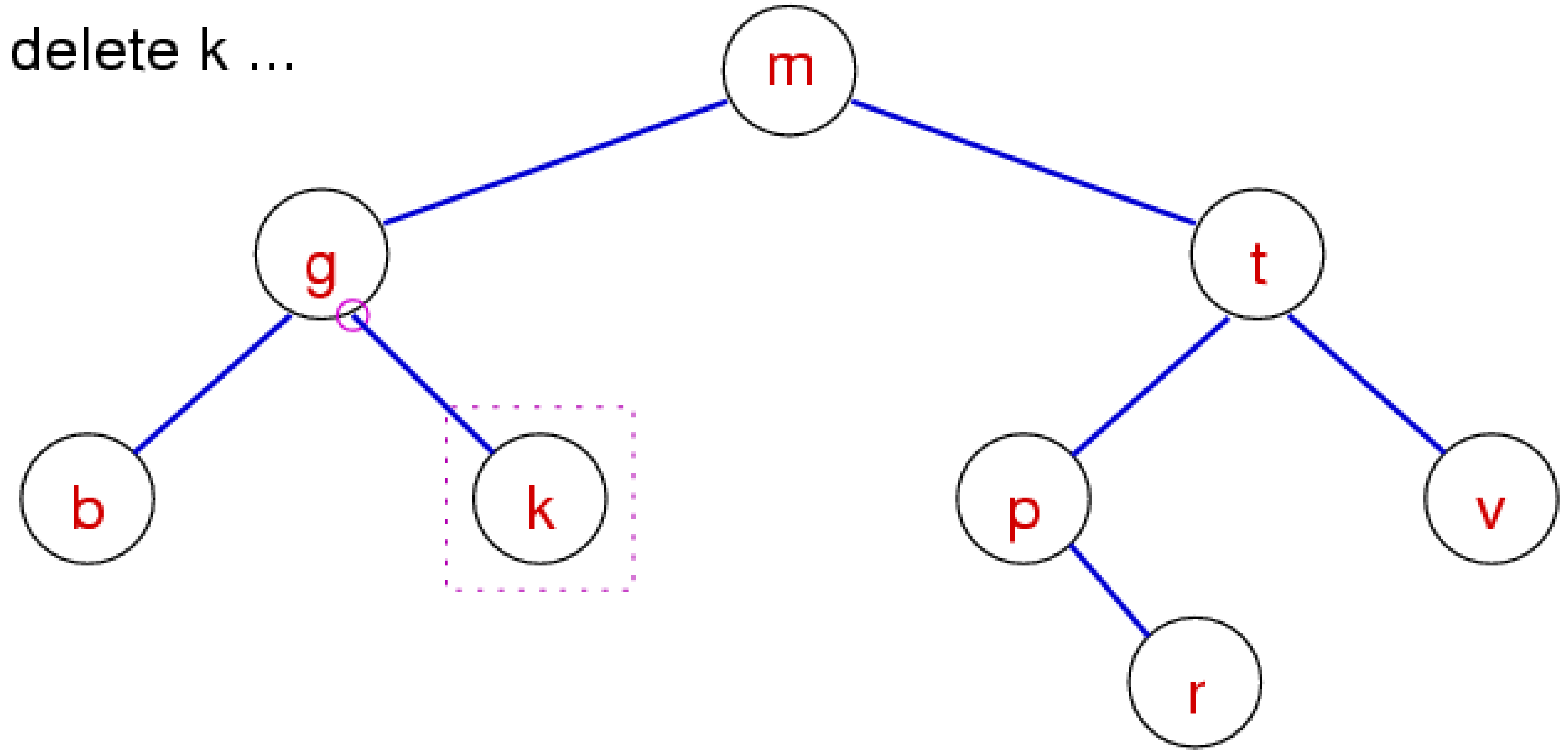
- Insertion into a binary search tree is easy:
  - find location in tree where node to be added
  - create node and link to parent
- Deletion from a binary search tree is harder:
  - find the node to be deleted and its parent
  - unlink node from parent and delete
  - replace node in tree by ... ???

Deletion from BSTs…

- Easy option ... don't delete; just mark node as deleted
  - future searches simply ignore marked nodes
- If we want to delete, three cases to consider ...
  - zero subtrees ... unlink node from parent
  - one subtree ... replace node by child
  - two subtrees ... two children; one link in parent

Deletion from BSTs

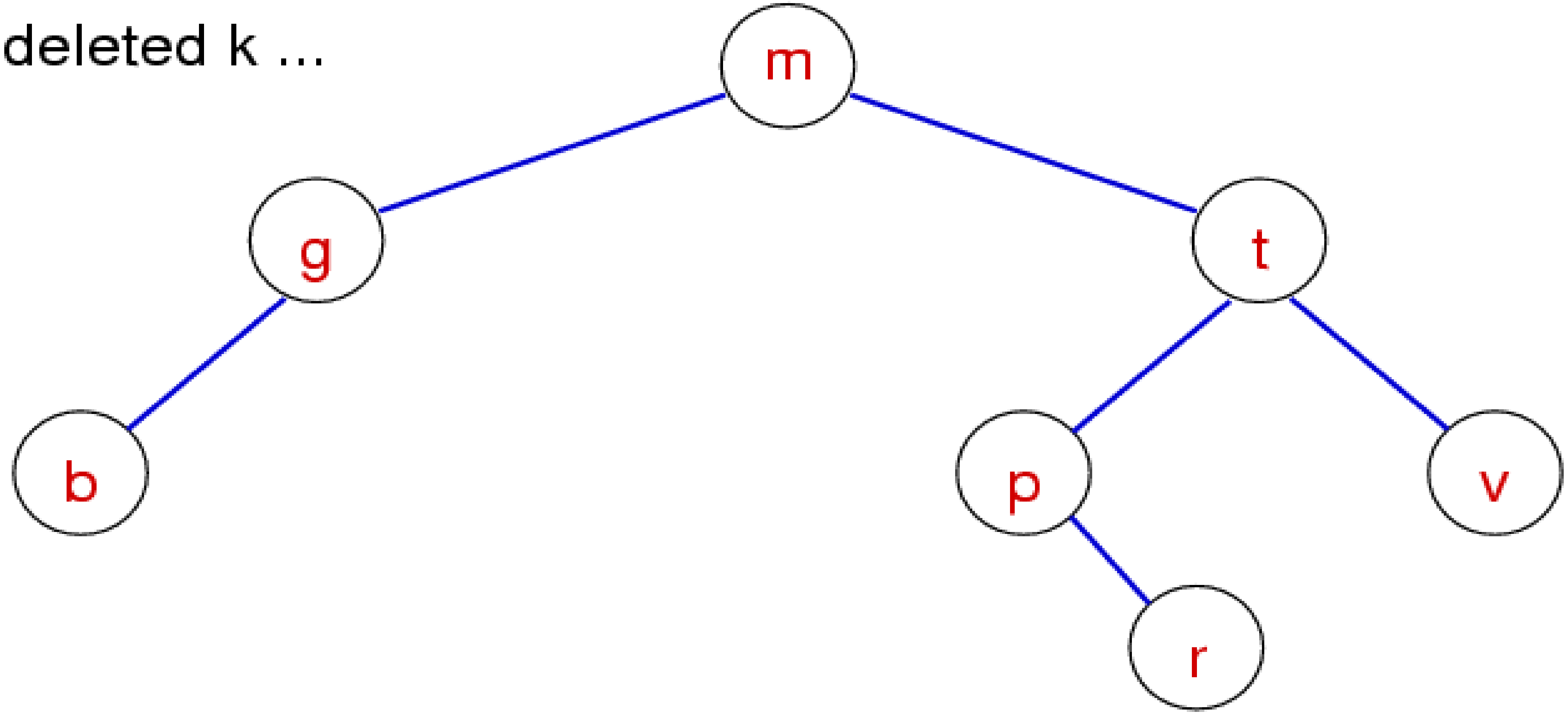
- Case 1: value to be deleted is a leaf (zero subtrees)



Deletion from BSTs

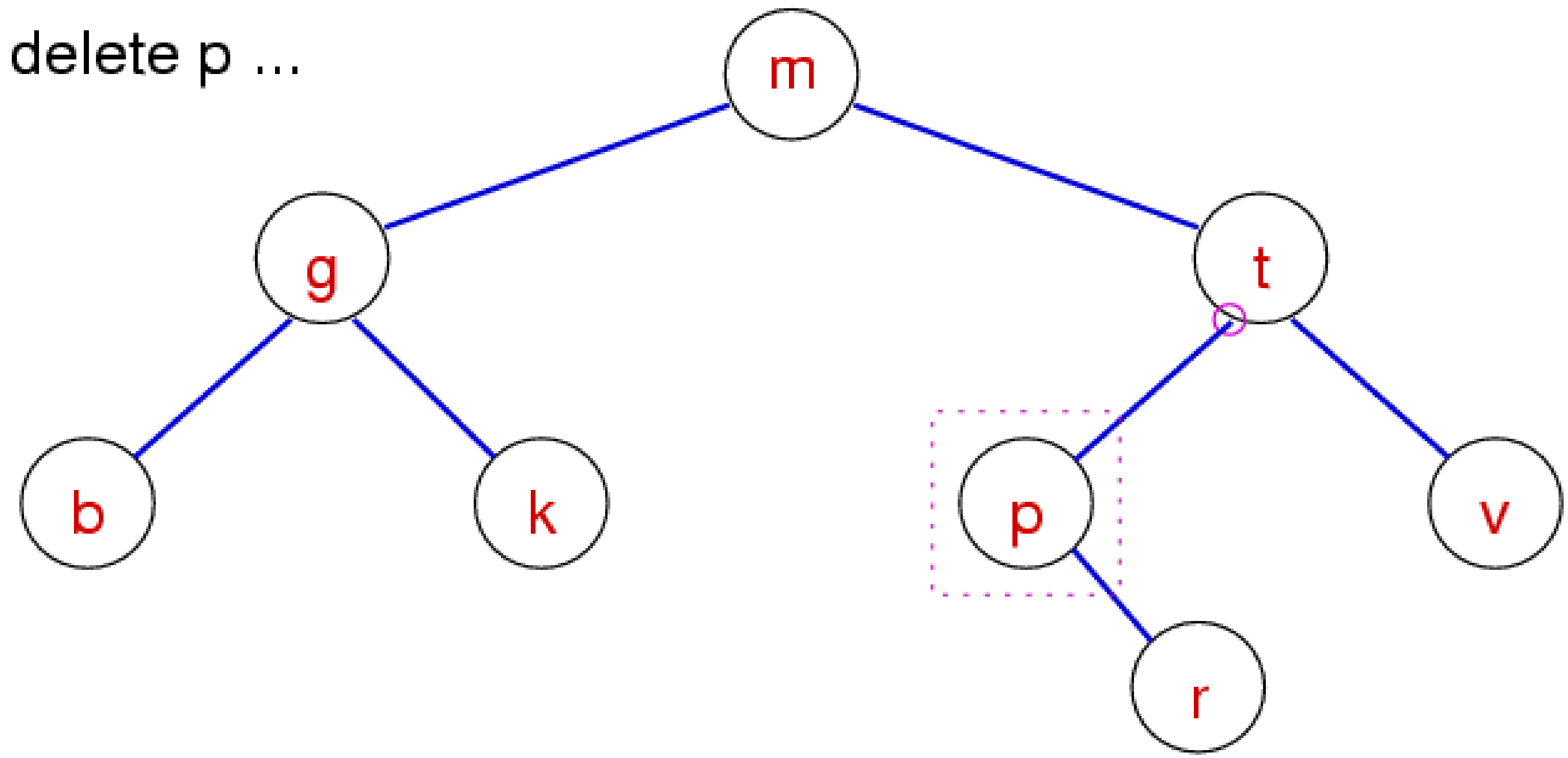
- Case 1: value to be deleted is a leaf (zero subtrees)

deleted k ...



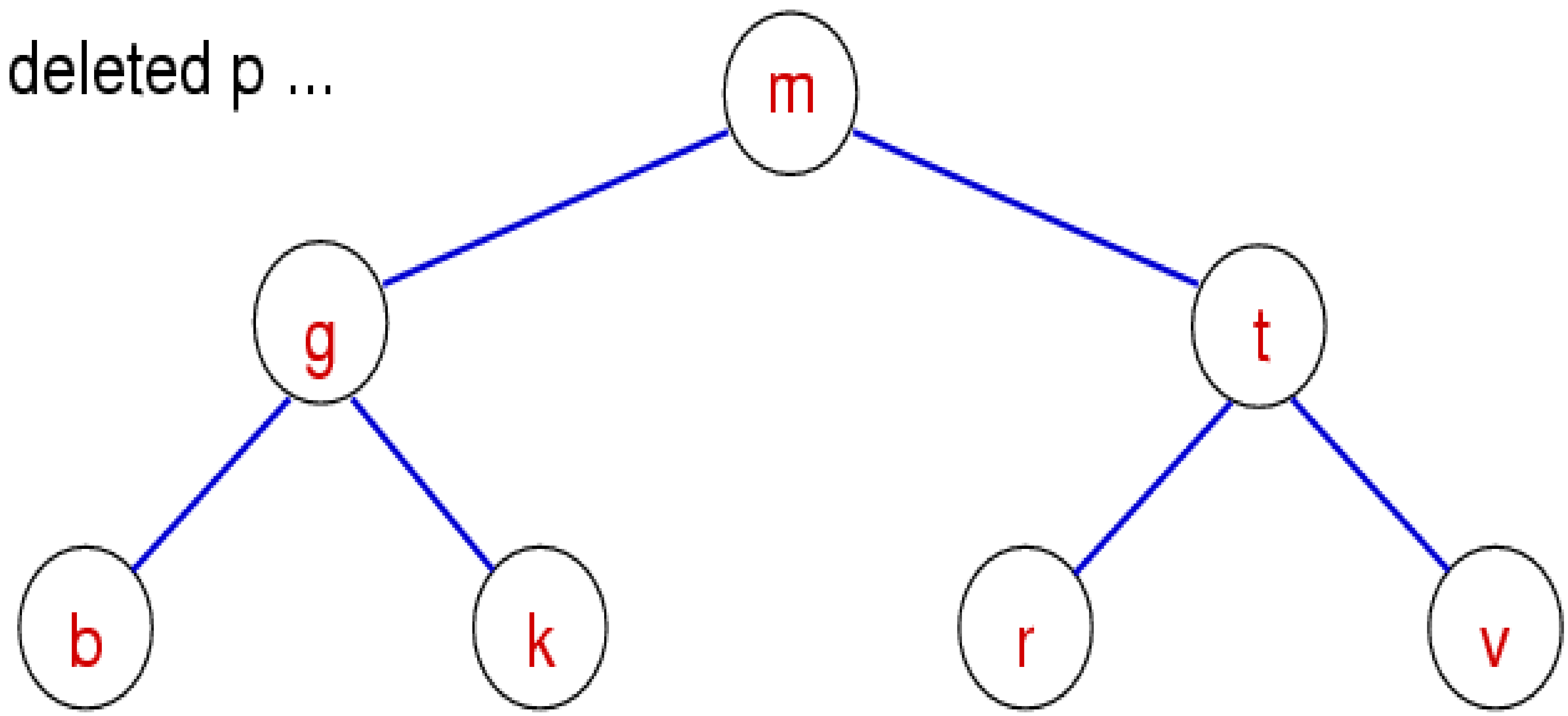
Deletion from BSTs

- Case 2: value to be deleted has one subtree



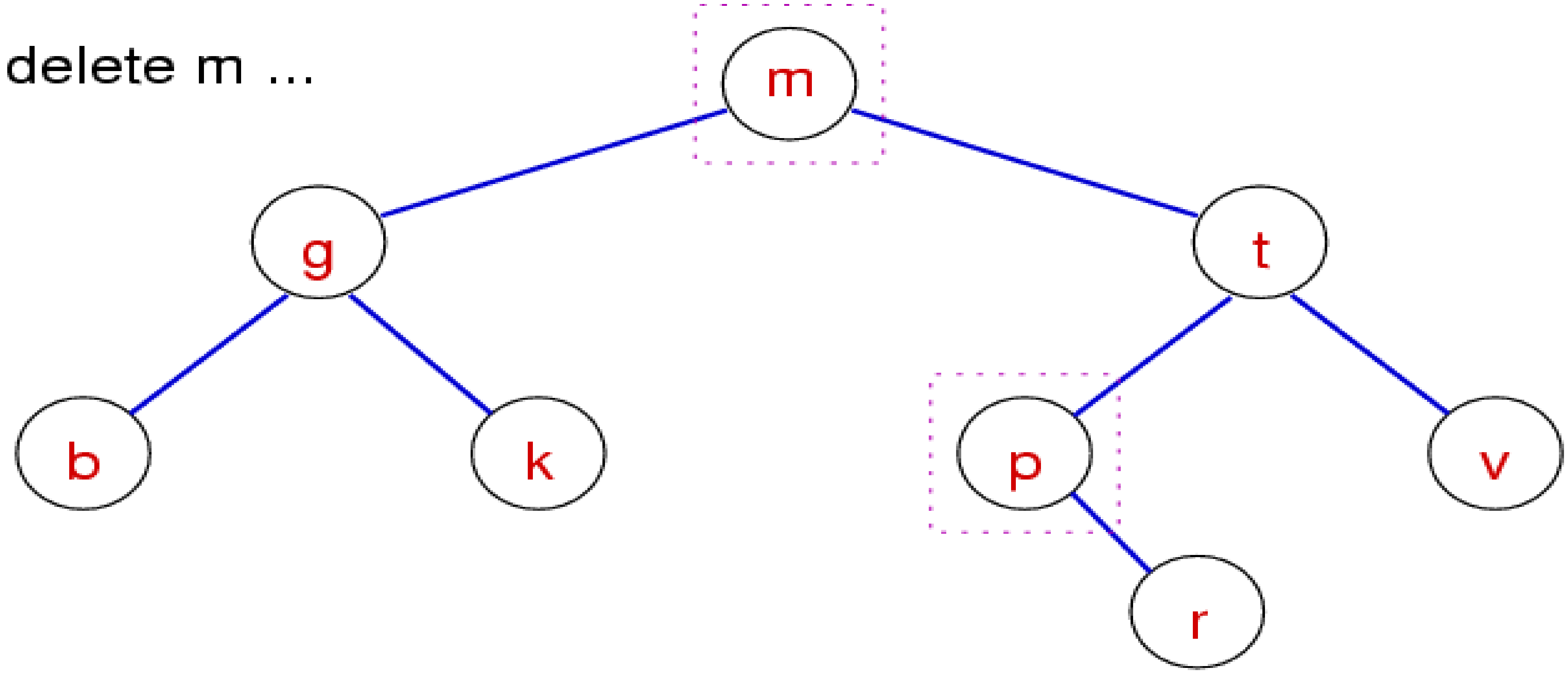
Deletion from BSTs

- Case 2: value to be deleted has one subtree



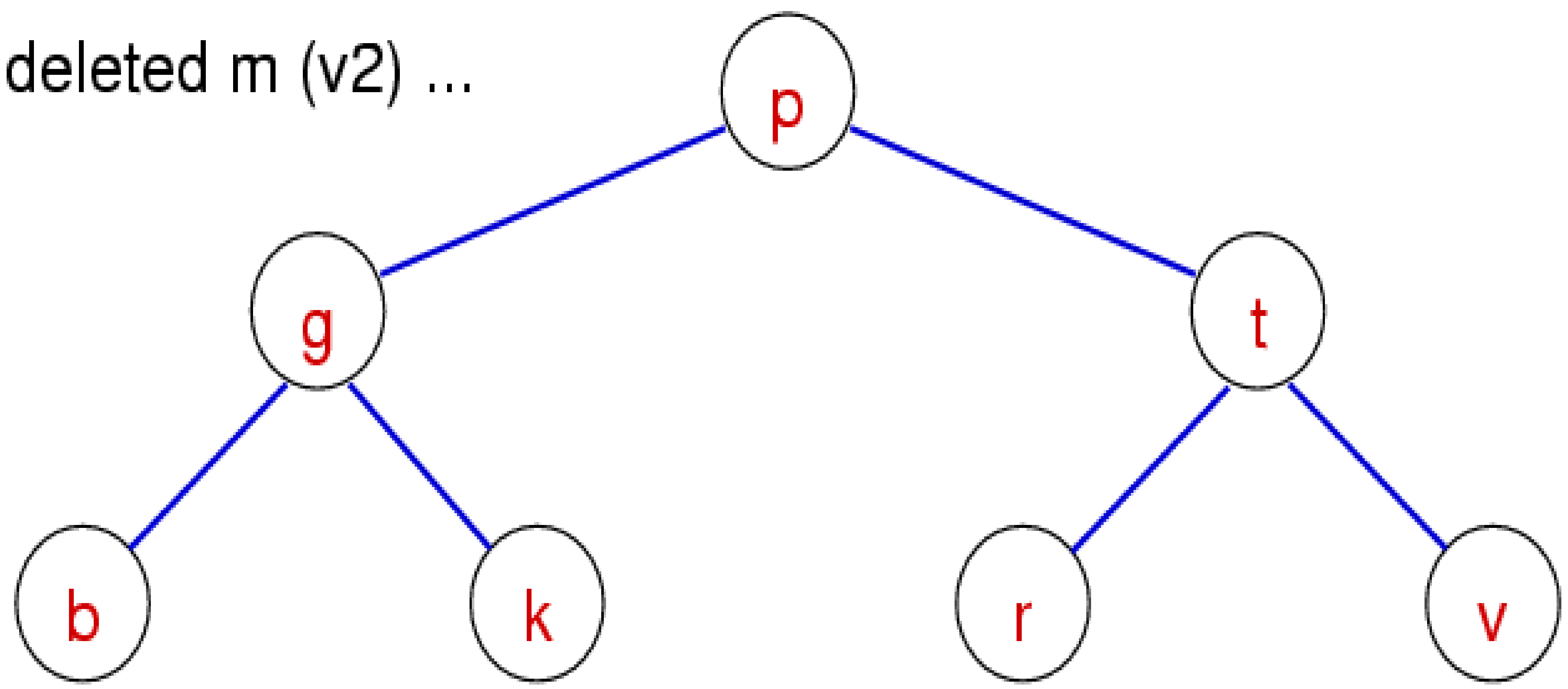
Deletion from BSTs

- Case 3a: value to be deleted has two subtrees
- Replace deleted node by its immediate successor
  - The smallest (leftmost) node in the right subtree



Deletion from BSTs

- Case : value to be deleted has two subtrees



Binary Search Tree Properties

- Cost for searching/deleting:
  - Worst case: key is not in BST – search the height of the tree
    - Balanced trees – $O(\log_2 n)$
    - Degenerate trees – $O(n)$
- Cost for insertion:
  - Always traverse the height of the tree
    - Balanced trees – $O(\log_2 n)$
    - Degenerate trees – $O(n)$

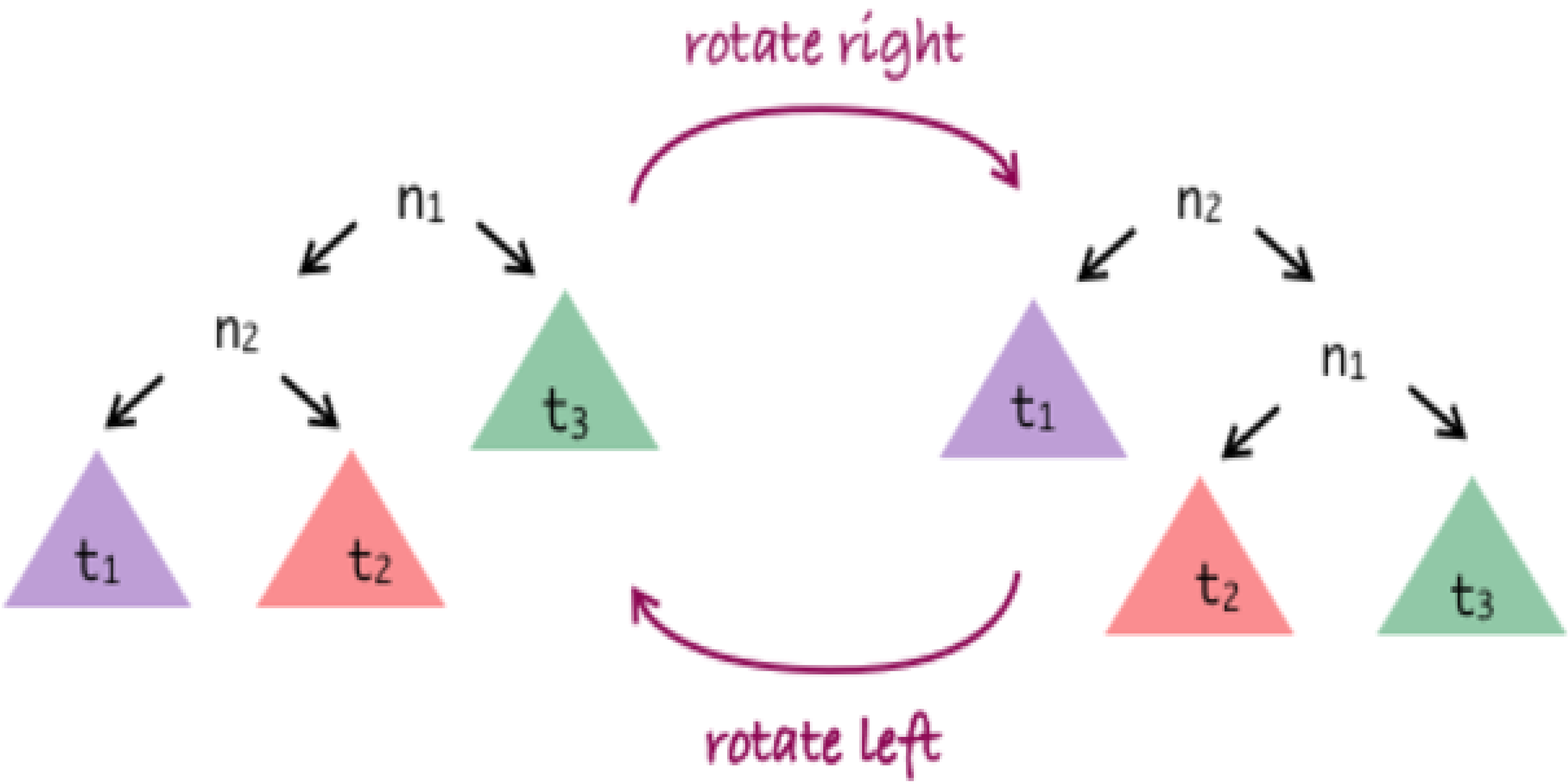
New Binary Search Tree

```
// Item, Key, Node, Link, Tree types as before #define
key(it) ((it).key)
// operations on keys
#define cmp(k1,k2) ((k1) - (k2))
#define lt(k1,k2) (cmp(k1,k2) < 0)
#define eq(k1,k2) (cmp(k1,k2) == 0)
#define gt(k1,k2) (cmp(k1,k2) > 0)
// standard tree operations
Tree newTree();
Tree insert(Tree, Item);
Tree delete(Tree, Key);
int find(Tree, Key);
```

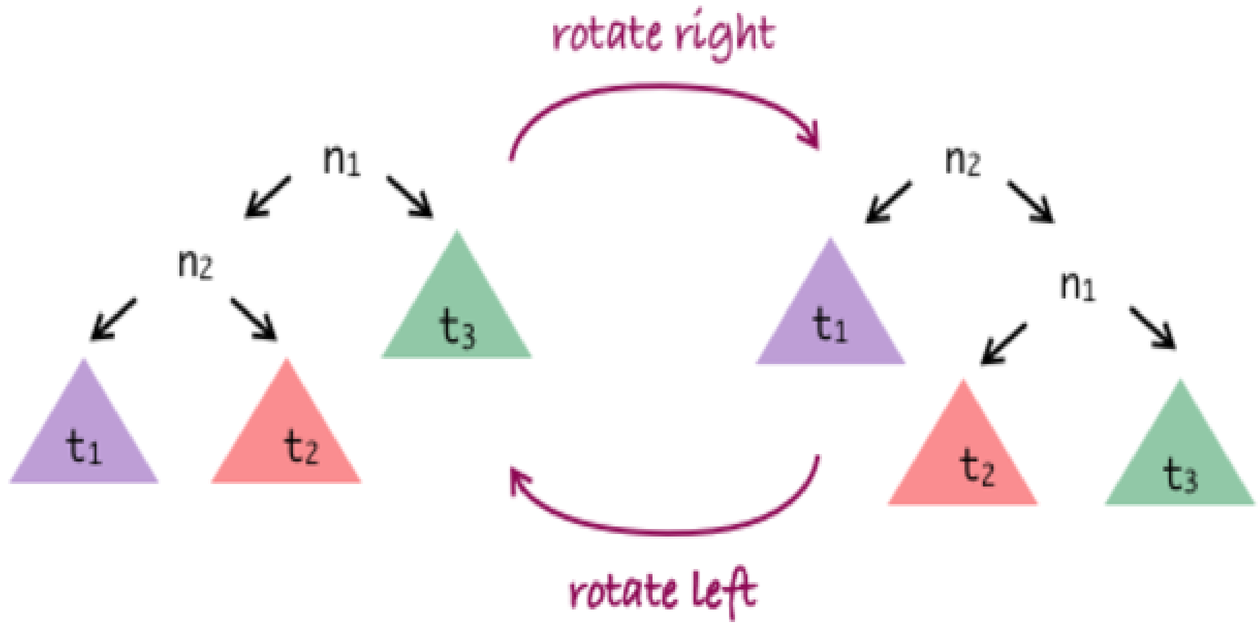
Insertion Using new Tree ADT

```
// more standard tree operations
void dropTree(Tree);
void showTree(Tree);
int depth(Tree);
int nnodes(Tree); // aka size()
// functions internal to ADT
Link rotateR(Link);
Link rotateL(Link);
Tree insertAtRoot(Tree, Item);
Tree insertRandom(Tree, Item);
```

Insert At Root - Rotate Operations



Insert At Root - Rotate Operations



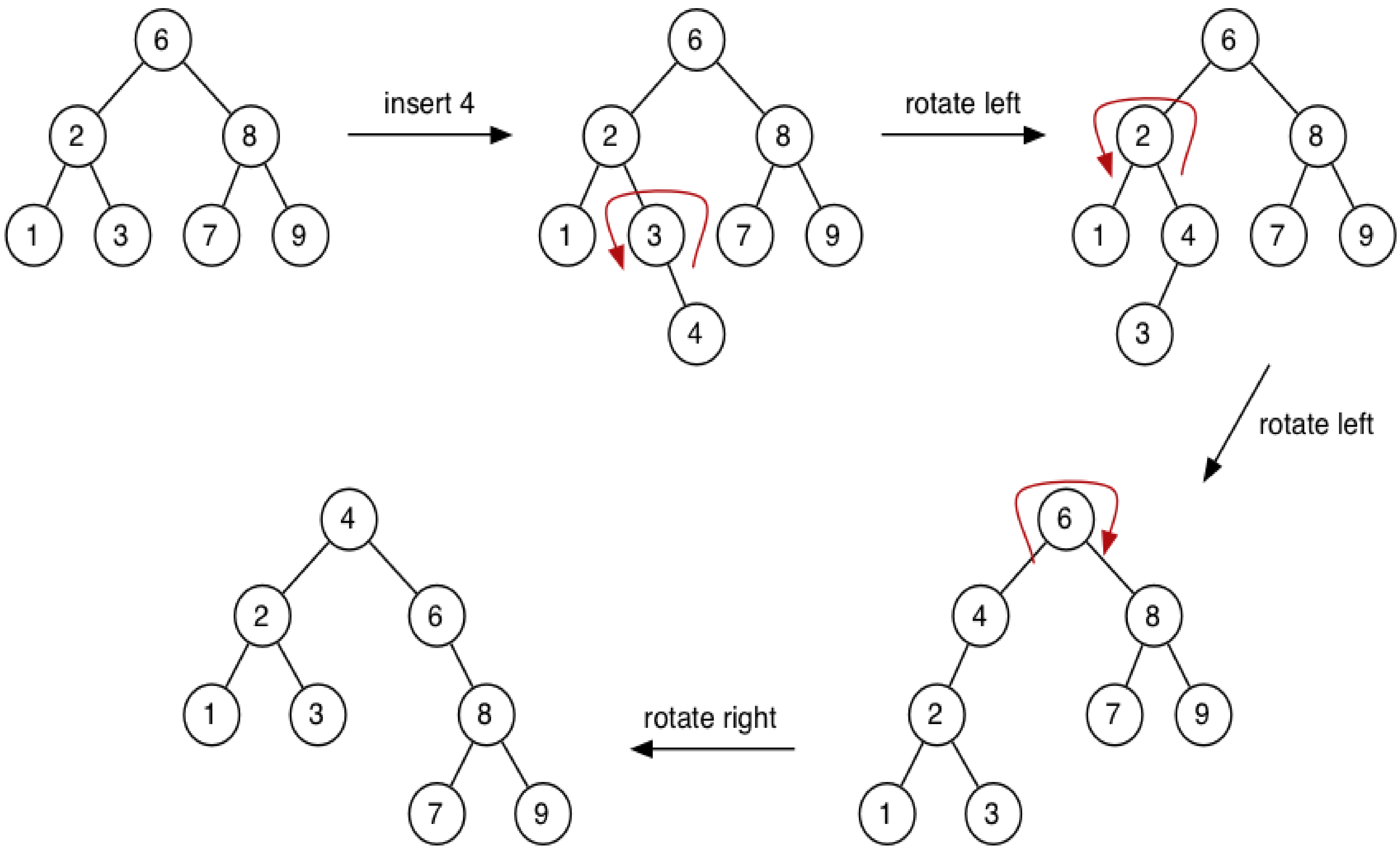

```
Link rotateR(Link n1) {
    if (n1 == NULL) return NULL;
    Link n2 = n1->left;
    if (n2 == NULL) return n1;
    n1->left = n2->right;
    n2->right = n1;
    return n2;
}
```

Left rotation is similar with **`n1`**/**`n2`** and left/right switched

Insertion At Root

- Previous description of BSTs inserted at leaves.
- Different approach: insert new value at root.
- Method for inserting at root (recursive):
- base case:
  - tree is empty; make new node and make it root
- recursive case:
  - insert new node as root of L/R subtree
  - lift new node to root by R/L rotation

Insertion At Root



Insertion At Root (Cont)

```
Tree insertAtRoot(Tree t, Item it) {
    if (t == NULL) return newNode(item);
    int diff = cmp(key(it), key(t->value));
    if (diff == 0) t->value = it;
    else if (diff < 0)
    {     t->left = insertAtRoot(t->left, it);
          t = rotateR(t);
     }
     else if (diff > 0) {
          t->right = insertAtRoot(t->right, it);
          t = rotateL(t);
     }
return t;
}
```