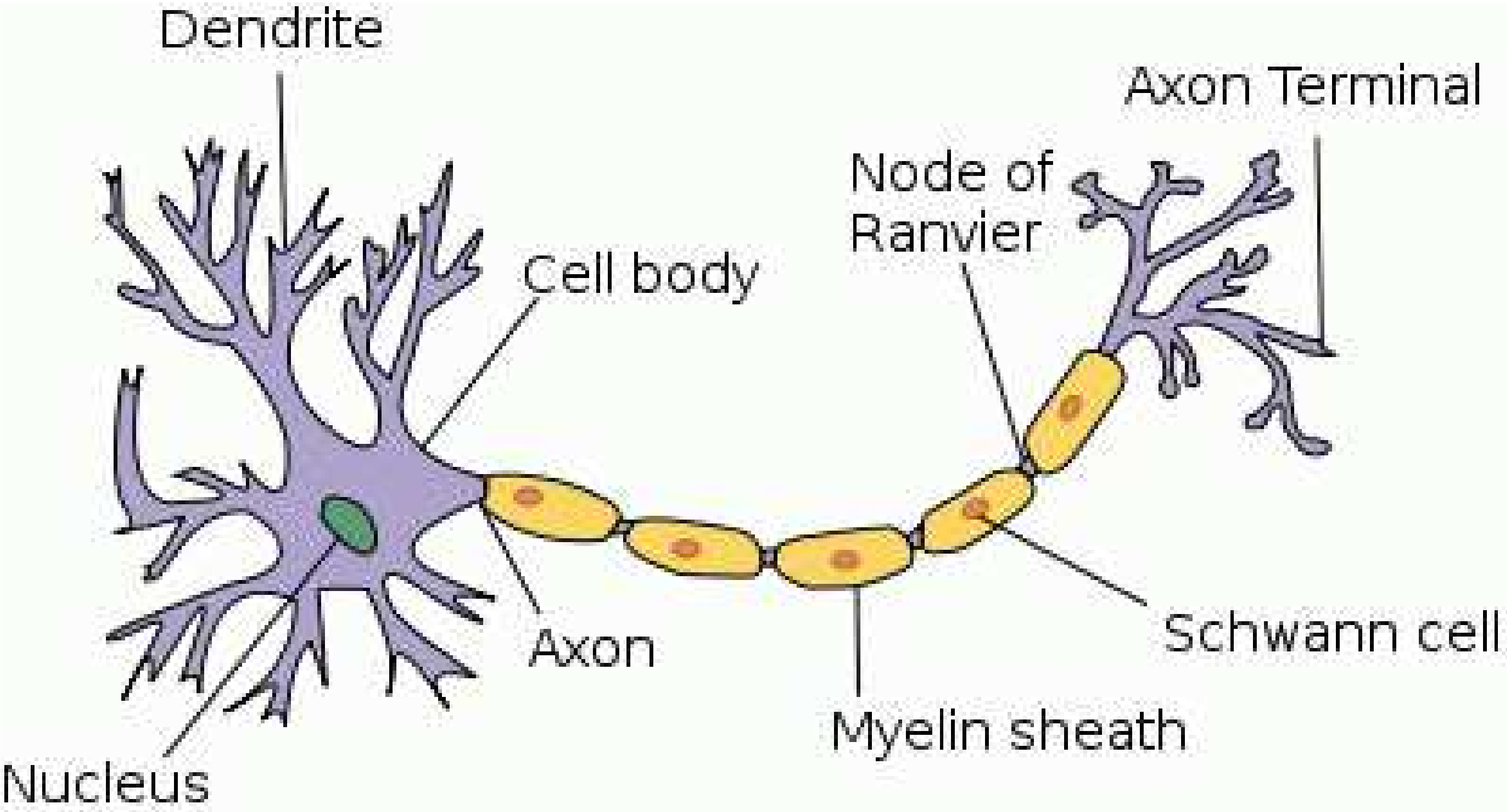
COMP9444
Neural Networks and Deep Learning

2. Perceptrons

Outline

- Neurons – Biological and Artificial
- Perceptron Learning
- Linear Separability
- Multi-Layer Networks

Structure of a Typical Neuron



Biological Neurons

The brain is made up of neurons (nerve cells) which have

- a cell body (soma)
- dendrites (inputs)
- an axon (outputs)
- synapses (connections between cells)

Synapses can be exitatory or inhibitory and may change over time.

When the inputs reach some threshhold an action potential (electrical pulse) is sent along the axon to the outputs.

Artificial Neural Networks

(Artificial) Neural Networks are made up of nodes which have

- inputs edges, each with some weight
- outputs edges (with weights)
- an activation level (a function of the inputs)

Weights can be positive or negative and may change over time (learning).

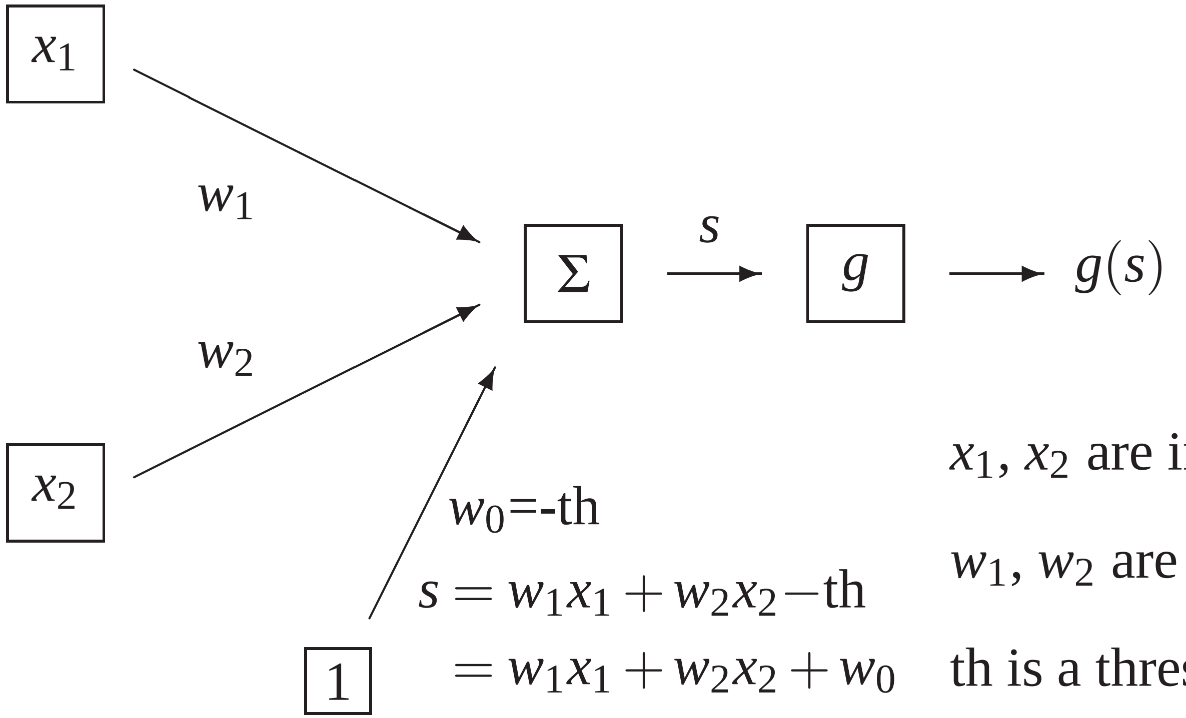
The input function is the weighted sum of the activation levels of inputs.

The activation level is a non-linear transfer function g of this input:

$$\text{activation}_i = g(s_i) = g(\sum_j w_{ij} x_j)$$

Some nodes are inputs (sensing), some are outputs (action)

McCulloch & Pitts Model of a Single Neuron



x_1, x_2 are inputs

w_1, w_2 are synaptic weights

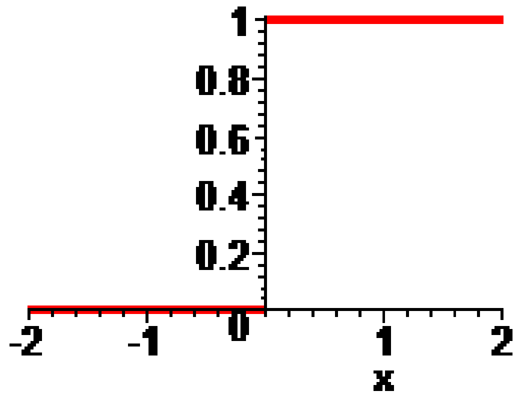
th is a threshold

w_0 is a **bias** weight

g is transfer function

Transfer function

Originally, a (discontinuous) step function was used for the transfer function:

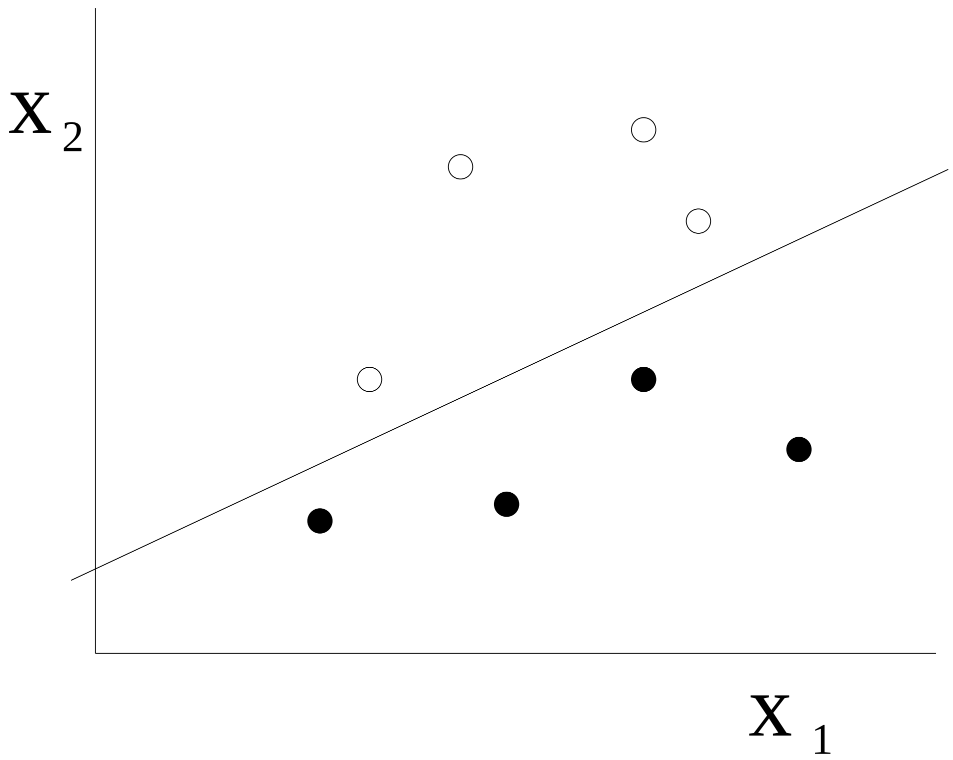


$$g(s) = \begin{cases} 1, & \text{if} \quad s \geq 0 \\ 0, & \text{if} \quad s < 0 \end{cases}$$

(Later, other transfer functions were introduced, which are continuous and smooth)

Linear Separability

Question: what kind of functions can a perceptron compute?



Answer: linearly separable functions

Linear Separability

Examples of linearly separable functions:

AND	$w_1 = w_2 = 1.0, \quad w_0 = -1.5$
OR	$w_1 = w_2 = 1.0, \quad w_0 = -0.5$
NOR	$w_1 = w_2 = -1.0, \quad w_0 = 0.5$

Q: How can we train it to learn a new function?

Rosenblatt Perceptron

Perceptron Learning Rule

Adjust the weights as each input is presented.

recall: $s = w_1 x_1 + w_2 x_2 + w_0$

if $g(s) = 0$ but should be 1,

$$
\begin{aligned}
w_k &\leftarrow w_k + \eta\, x_k \\
w_0 &\leftarrow w_0 + \eta
\end{aligned}
$$

$$
\text{so} \quad s \leftarrow s + \eta\,(1 + \sum_k x_k^2\,)
$$

if $g(s) = 1$ but should be 0,

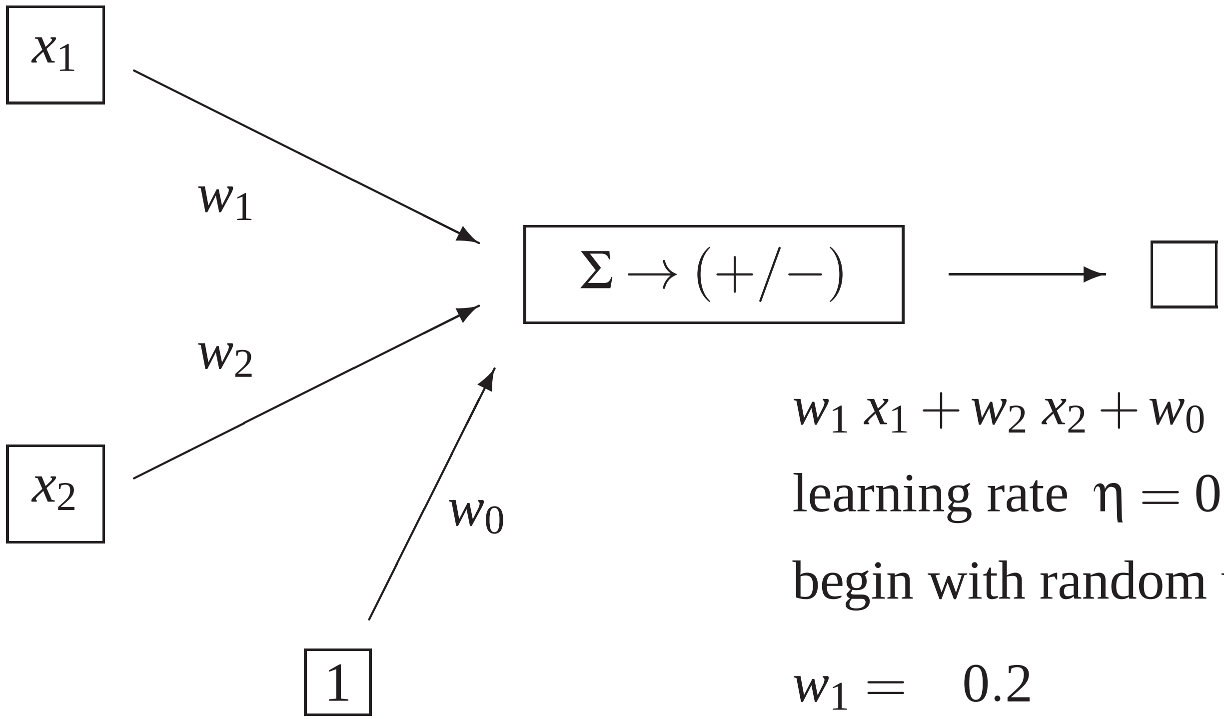
$$
\begin{aligned}
w_k &\leftarrow w_k - \eta\, x_k \\
w_0 &\leftarrow w_0 - \eta
\end{aligned}
$$

$$
\text{so} \quad s \leftarrow s - \eta\,(1 + \sum_k x_k^2\,)
$$

otherwise, weights are unchanged. ($\eta > 0$ is called the **learning rate**)

Theorem: This will eventually learn to classify the data correctly, as long as they are **linearly separable**.

Perceptron Learning Example



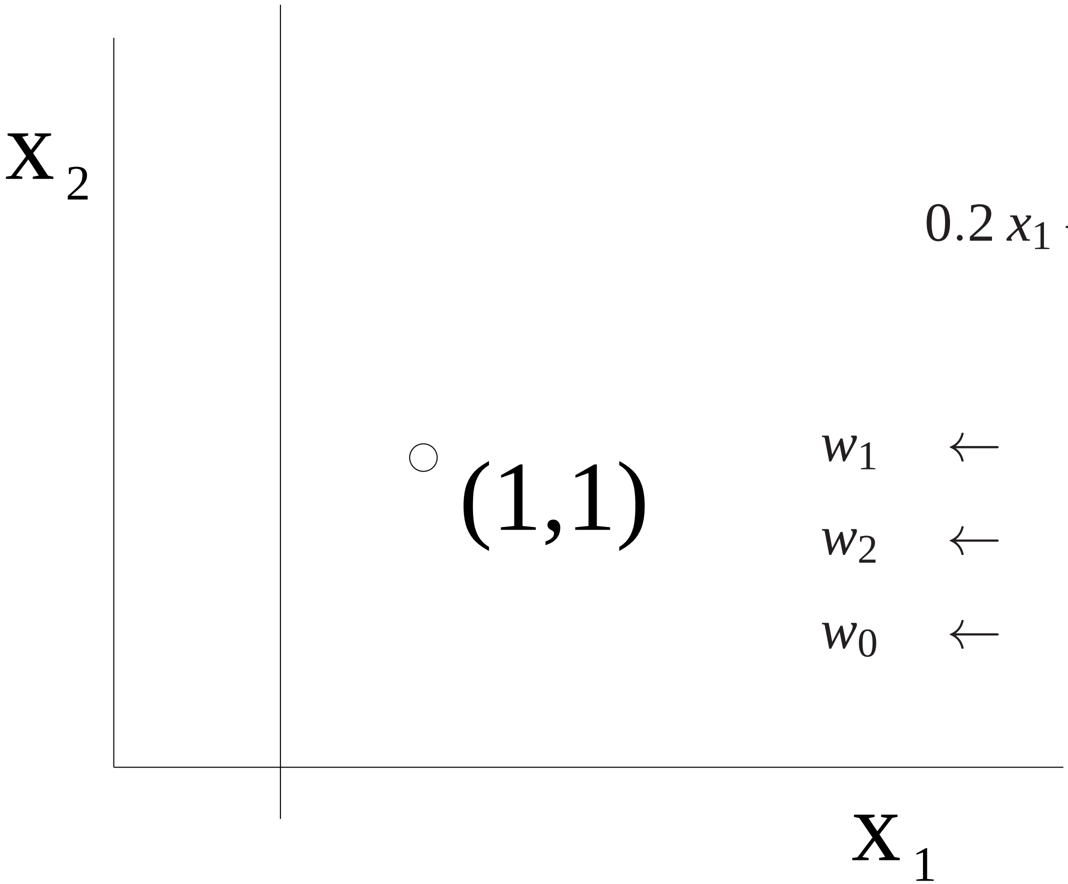
$w_1\, x_1 + w_2\, x_2 + w_0 > 0$

learning rate $\eta = 0.1$

begin with random weights

$$\begin{aligned} w_1 &= 0.2 \\ w_2 &= 0.0 \\ w_0 &= -0.1 \end{aligned}$$

Training Step 1



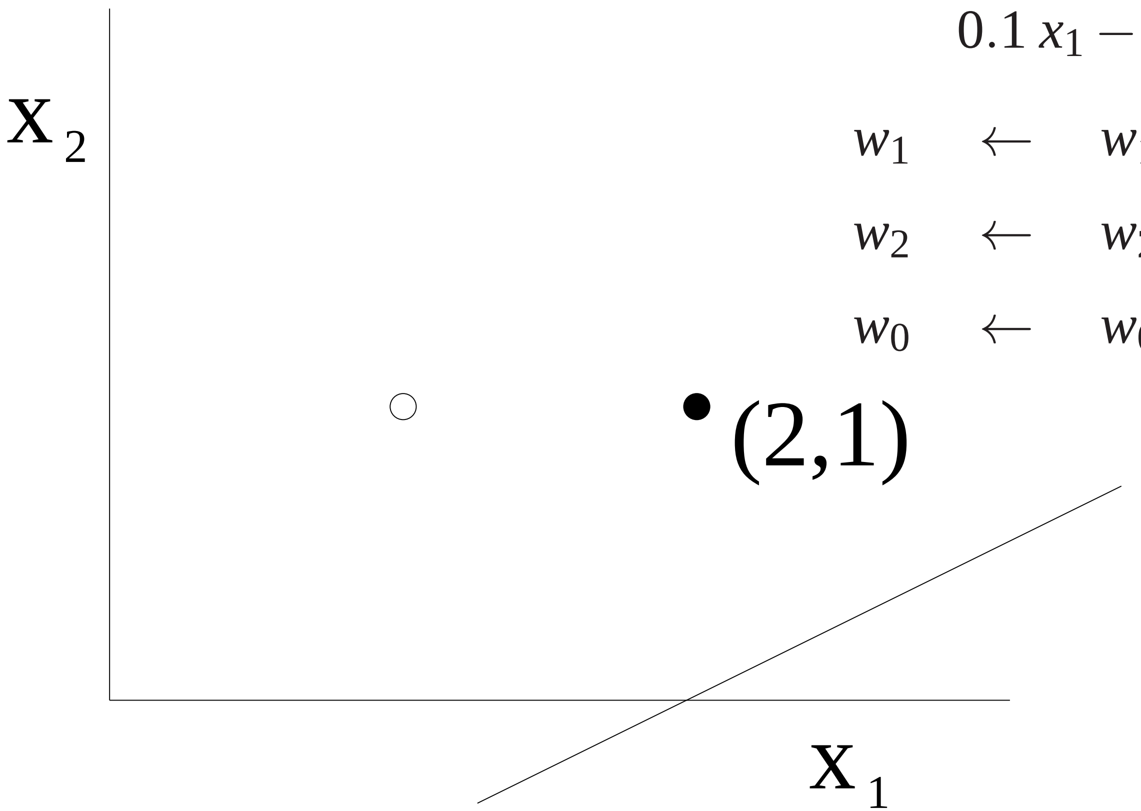
$$0.2\,x_1 + 0.0\,x_2 - 0.1 > 0$$

$$\begin{aligned}
w_1 &\leftarrow w_1 - \eta\, x_1 &= 0.1\\
w_2 &\leftarrow w_2 - \eta\, x_2 &= -0.1\\
w_0 &\leftarrow w_0 - \eta &= -0.2
\end{aligned}$$

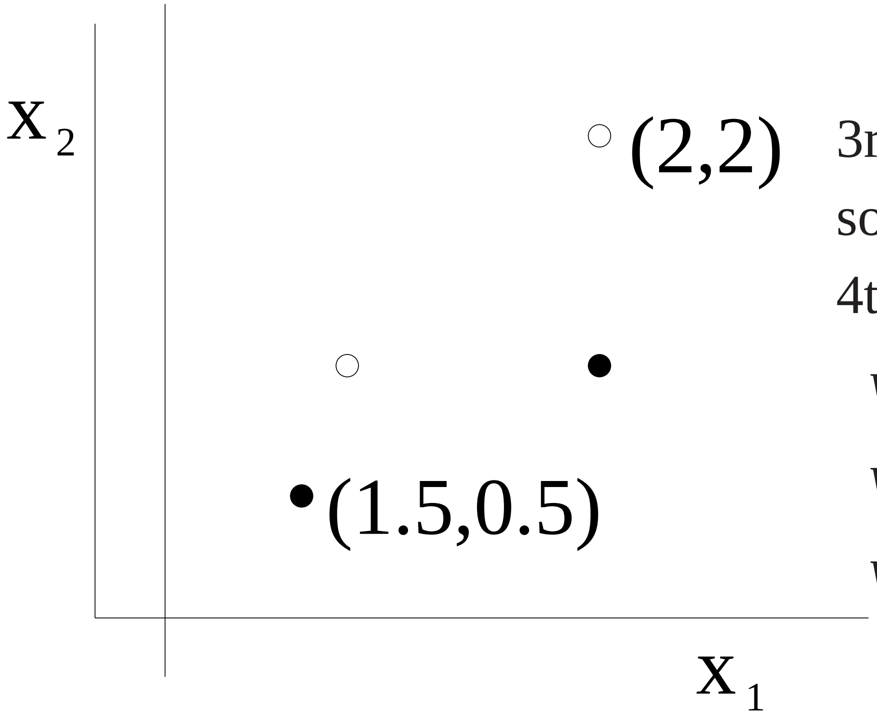
Training Step 2

$$0.1\,x_1 - 0.1\,x_2 - 0.2 > 0$$

$$\begin{aligned} w_1 &\leftarrow w_1 + \eta\, x_1 &&= 0.3 \\ w_2 &\leftarrow w_2 + \eta\, x_2 &&= 0.0 \\ w_0 &\leftarrow w_0 + \eta &&= -0.1 \end{aligned}$$



Training Step 3



$$0.3\,x_1 + 0.0\,x_2 - 0.1 > 0$$

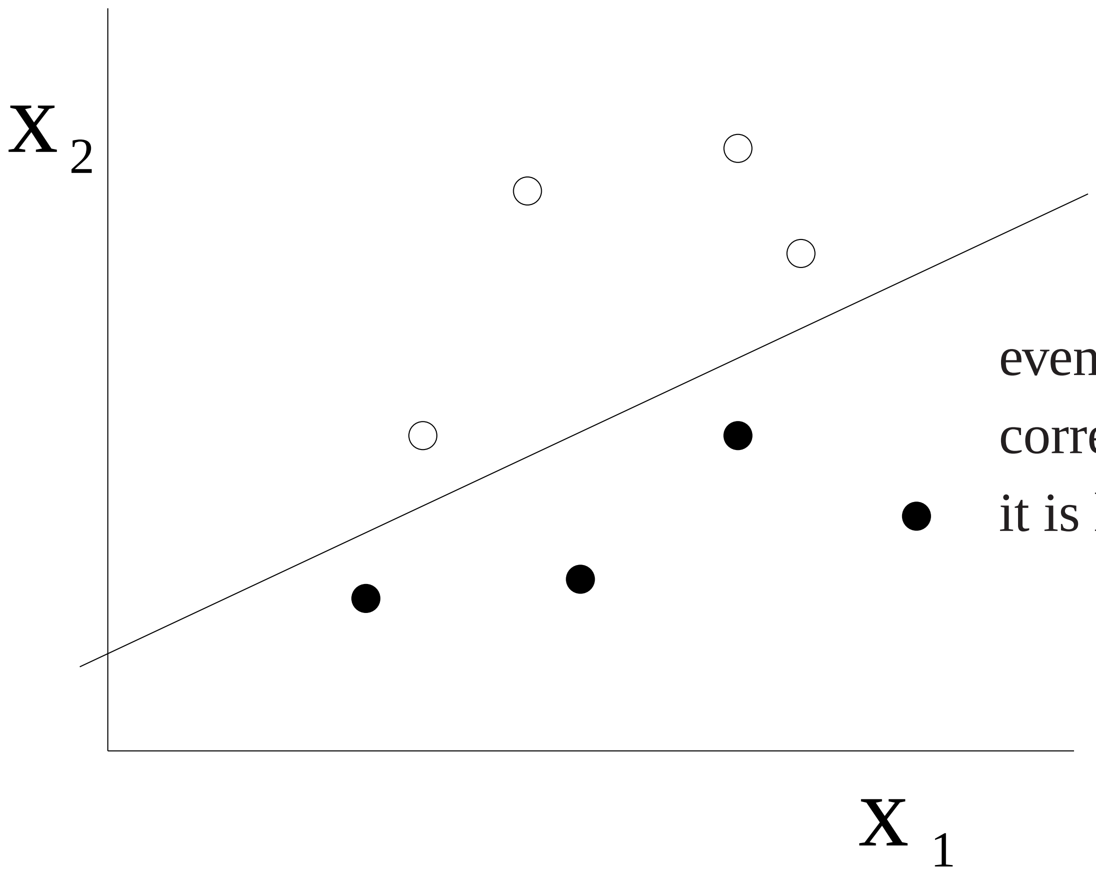
3rd point correctly classified, so no change

4th point:

$$\begin{aligned} w_1 &\leftarrow w_1 - \eta\, x_1 &= 0.1 \\ w_2 &\leftarrow w_2 - \eta\, x_2 &= -0.2 \\ w_0 &\leftarrow w_0 - \eta &= -0.2 \end{aligned}$$

$$0.1\,x_1 - 0.2\,x_2 - 0.2 > 0$$

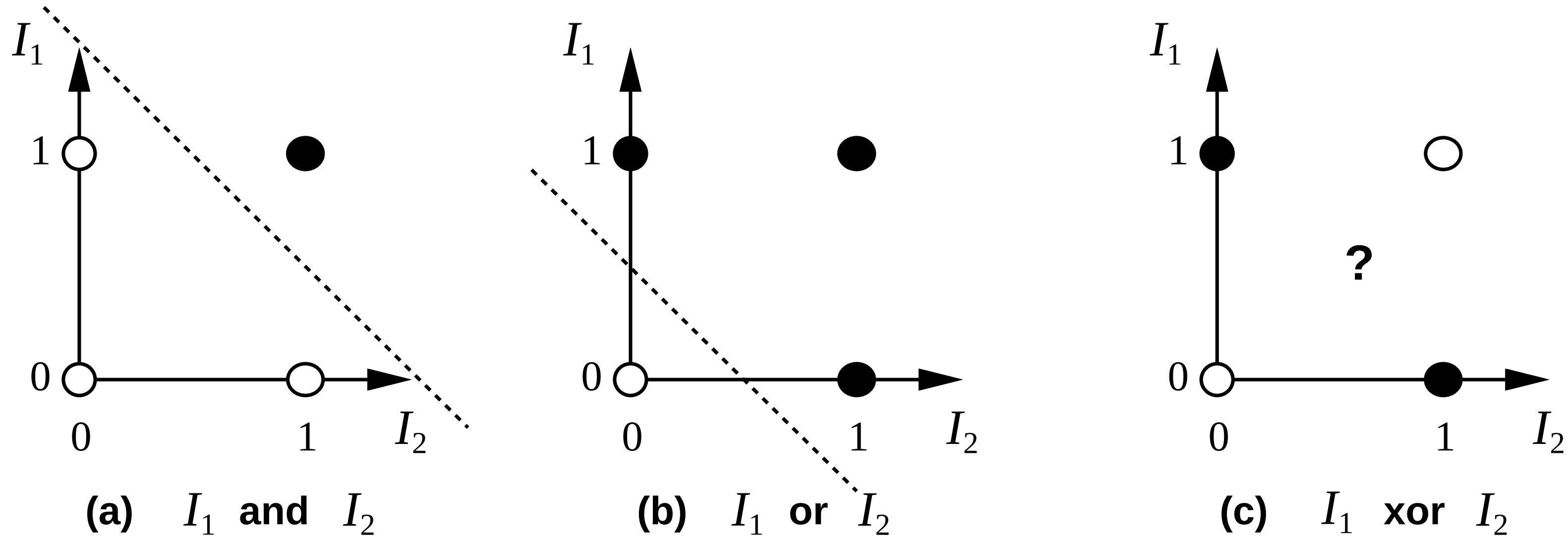
Final Outcome



eventually, all the data will be correctly classified (provided it is linearly separable)

Limitations of Perceptrons

Problem: many useful functions are not linearly separable (e.g. XOR)

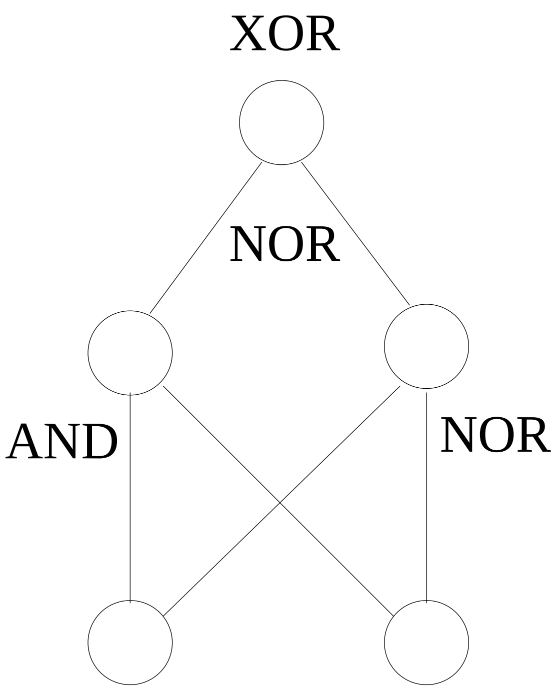


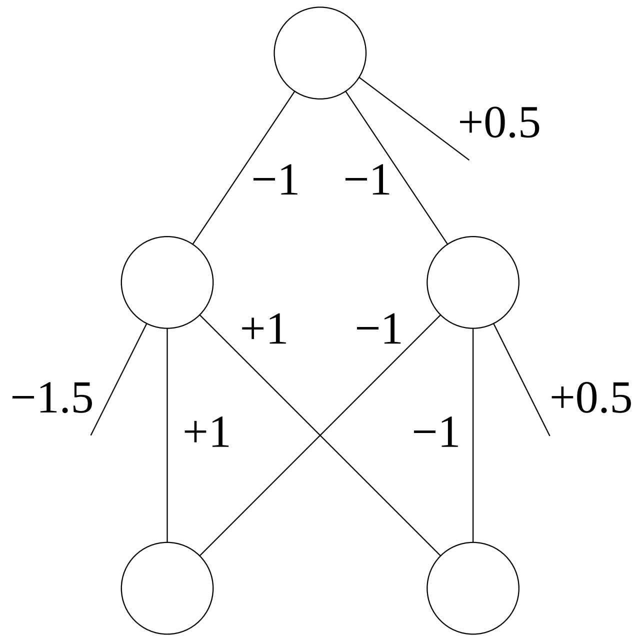
Possible solution:

x_1 XOR x_2 can be written as: (x_1 AND x_2) NOR (x_1 NOR x_2)

Recall that AND, OR and NOR can be implemented by perceptrons.

Multi-Layer Neural Networks





Problem: How can we train it to learn a new function? (credit assignment)

Historical Context

In 1969, Minsky and Papert published a book highlighting the limitations of Perceptrons, and lobbied various funding agencies to redirect funding away from neural network research, preferring instead logic-based methods such as expert systems.

It was known as far back as the 1960's that any given logical function could be implemented in a 2-layer neural network with step function activations. But, the the question of how to learn the weights of a multi-layer neural network based on training examples remained an open problem. The solution, which we describe in the next section, was found in 1976 by Paul Werbos, but did not become widely known until it was rediscovered in 1986 by Rumelhart, Hinton and Williams.