COMP9444
Neural Networks and Deep Learning

6. Recurrent Networks

Textbook, Chapter 10

Outline

- Processing Temporal Sequences
- Sliding Window
- Recurrent Network Architectures
- Hidden Unit Dynamics
- Long Short Term Memory

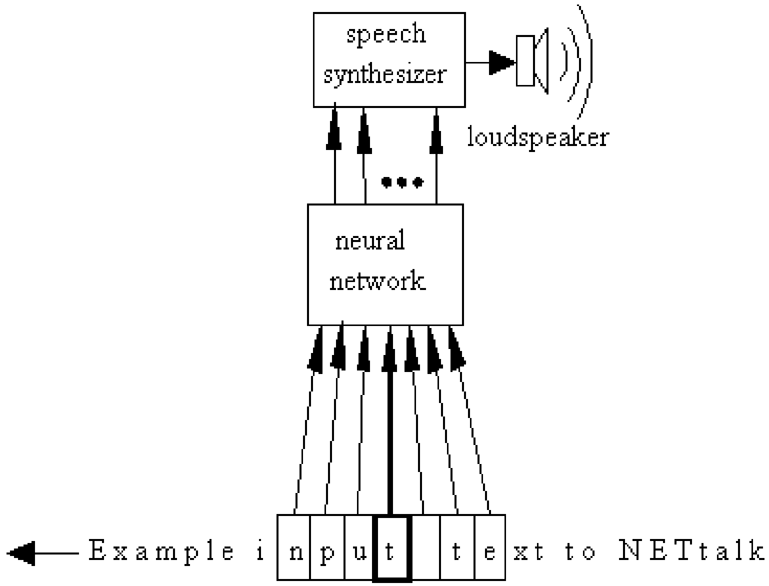
Processing Temporal Sequences

There are many tasks which require a sequence of inputs to be processed rather than a single input.

- speech recognition
- time series prediction
- machine translation
- handwriting recognition

How can neural network models be adapted for these tasks?

Sliding Window



The simplest way to feed temporal input to a neural network is the "sliding window" approach, first used in the NetTalk system (Sejnowski & Rosenberg, 1987).

NetTalk Task

Given a sequence of 7 characters, predict the phonetic pronunciation of the middle character.

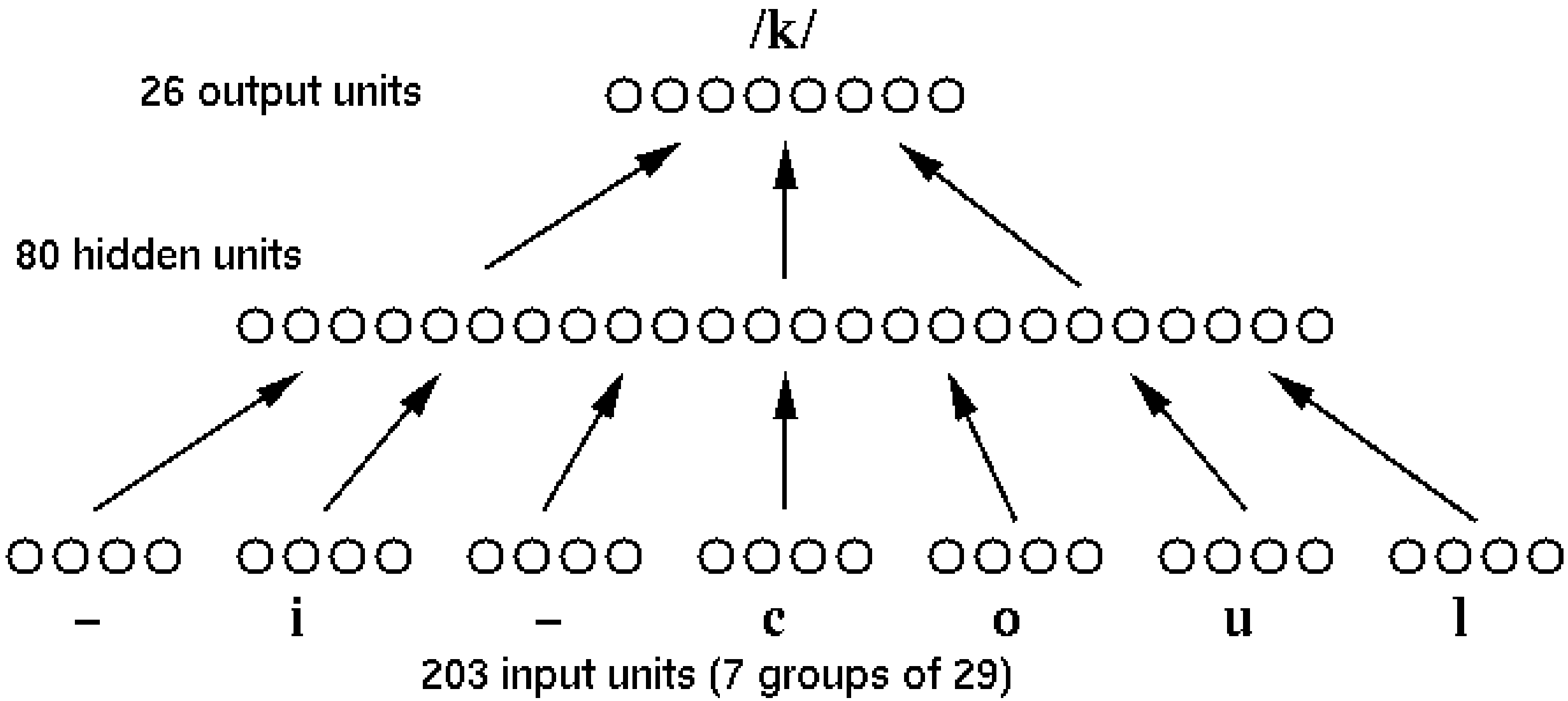
For this task, we need to know the characters on both sides.

For example, how are the vowels in these words pronounced?

pa pat pate paternal

mo mod mode modern

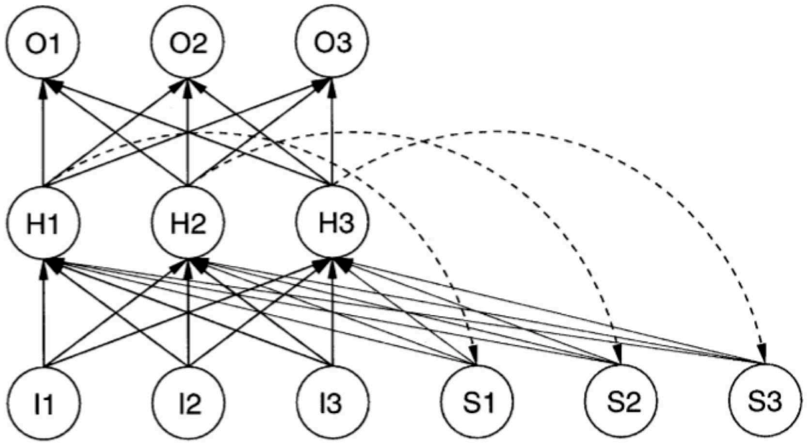
NetTalk Architecture



NetTalk

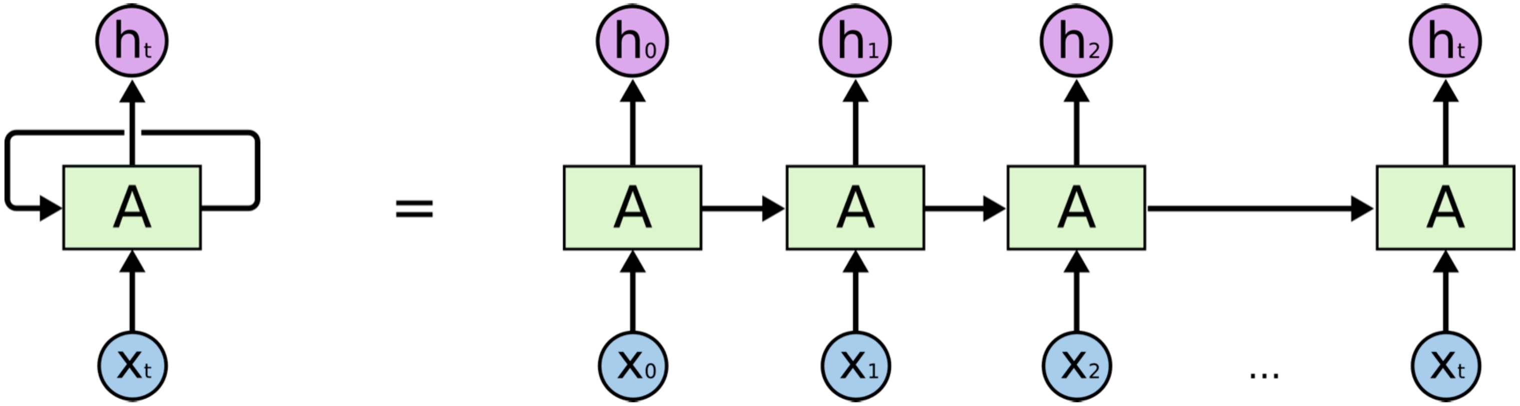
- NETtalk gained a lot of media attention at the time.
- Hooking it up to a speech synthesizer was very cute. In the early stages of training, it sounded like a babbling baby. When fully trained, it pronounced the words mostly correctly (but sounded somewhat robotic).
- Later studies on similar tasks have often found that a decision tree could produce equally good or better accuracy.
- This kind of approach can only learn short term dependencies, not the medium or long term dependencies that are required for some tasks.

Simple Recurrent Network (Elman, 1990)



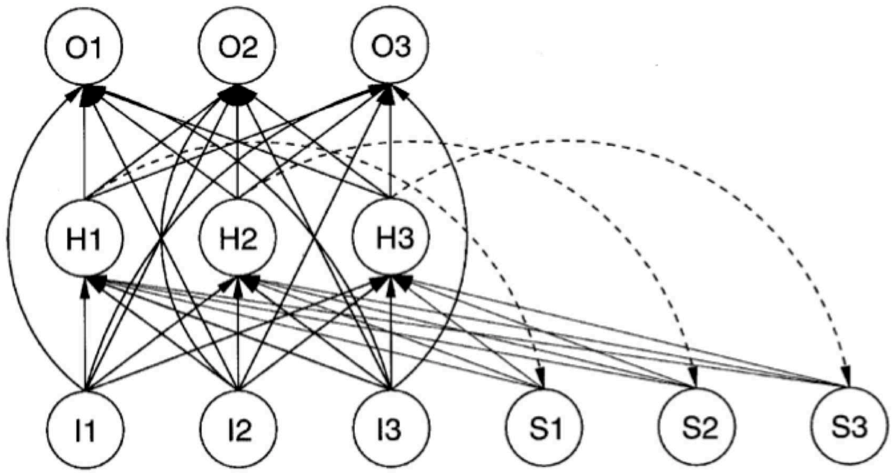
- at each time step, hidden layer activations are copied to “context” layer
- hidden layer receives connections from input and context layers
- the inputs are fed one at a time to the network, it uses the context layer to “remember” whatever information is required for it to produce the correct output

Back Propagation Through Time



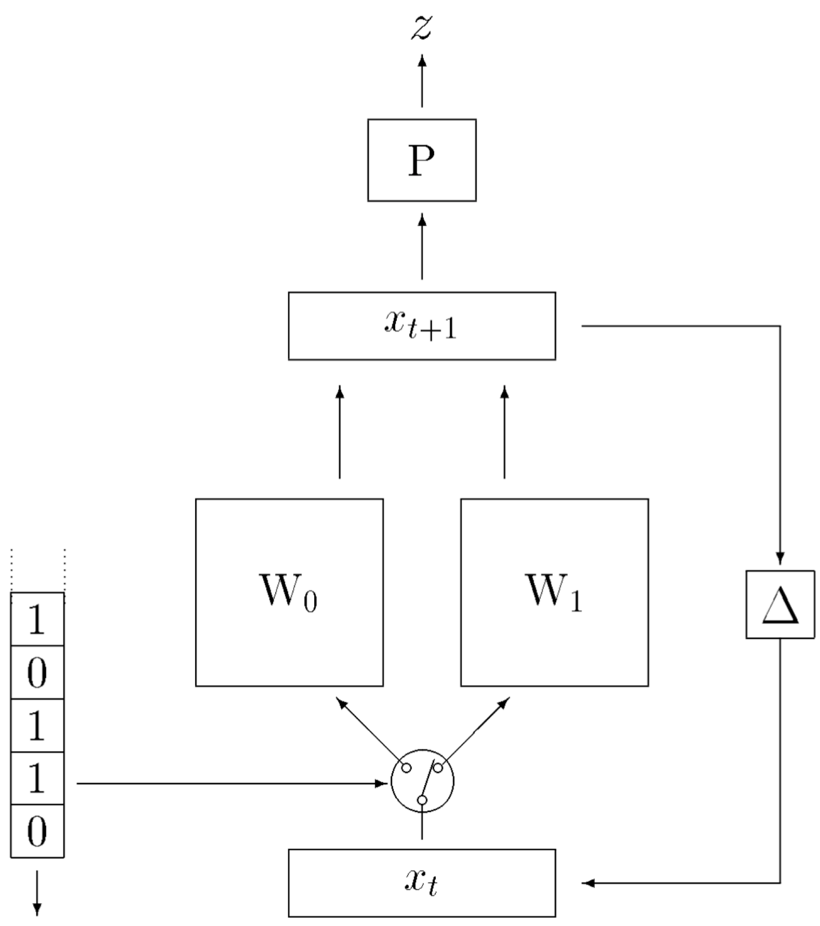
- we can "unroll" a recurrent architecture into an equivalent feedforward architecture, with shared weights
- applying backpropagation to the unrolled architecture is reffered to as "backpropagation through time"
- we can backpropagate just one timestep, or a fixed number of timesteps, or all the way back to beginning of the sequence

Other Recurrent Network Architectures



- it is sometimes beneficial to add “shortcut” connections directly from input to output
- connections from output back to hidden have also been explored (sometimes called “Jordan Networks”)

Second Order (or Gated) Networks



$$x_t^j = \tanh(W_{\sigma_t}^{j0} + \sum_{k=1}^{d} W_{\sigma_t}^{jk} x_{t-1}^k)$$

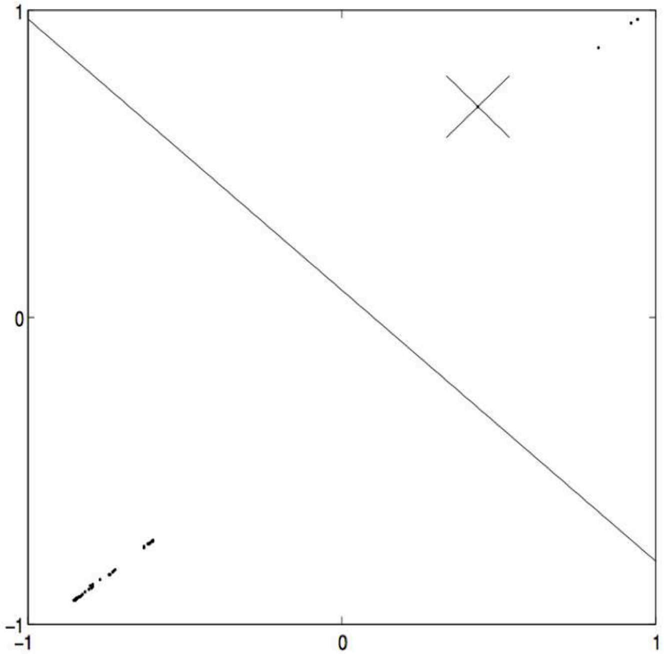
$$z = \tanh(P_0 + \sum_{j=1}^{d} P_j x_n^j)$$

Task: Formal Language Recognition

Accept	Reject
1	0
1 1	1 0
1 1 1	0 1
1 1 1 1	0 0
1 1 1 1 1	0 1 1
1 1 1 1 1 1	1 1 0
1 1 1 1 1 1 1	1 1 1 1 1 1 1 0
1 1 1 1 1 1 1 1	1 0 1 1 1 1 1 1

Scan a sequence of characters one at a time,
then classify the sequence as Accept or Reject.

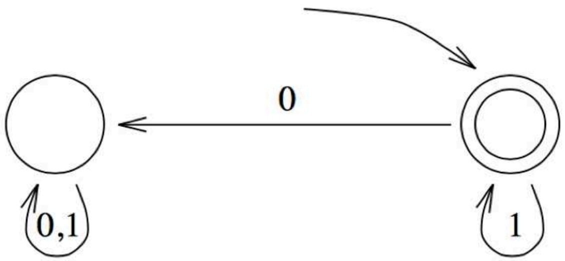
Dynamical Recognizers



$$W_0 = \begin{bmatrix} -0.89 & -0.09 & -0.14 \\ -1.13 & -0.09 & -0.14 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0.20 & 0.68 & 0.96 \\ 0.20 & 0.81 & 1.19 \end{bmatrix}$$

$$P = \begin{bmatrix} -0.07 & 0.66 & 0.75 \end{bmatrix}$$



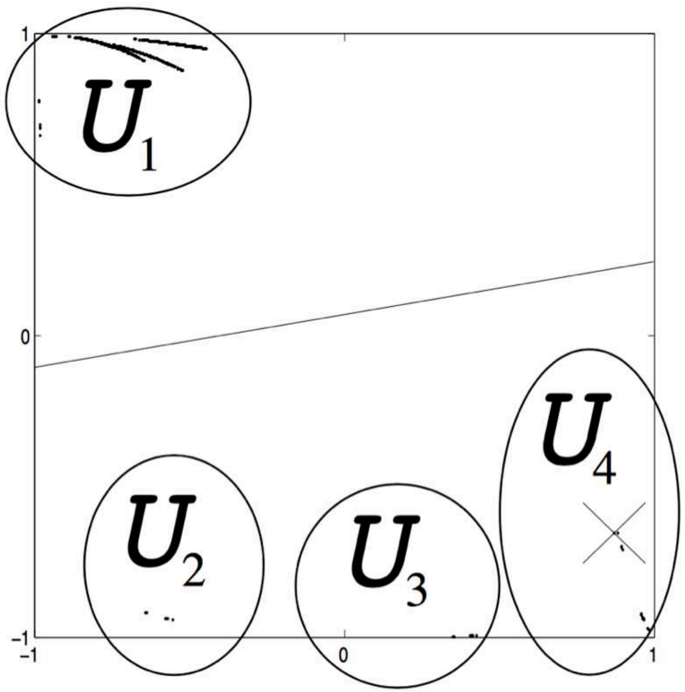
- gated network trained by BPTT
- emulates exactly the behaviour of Finite State Automaton

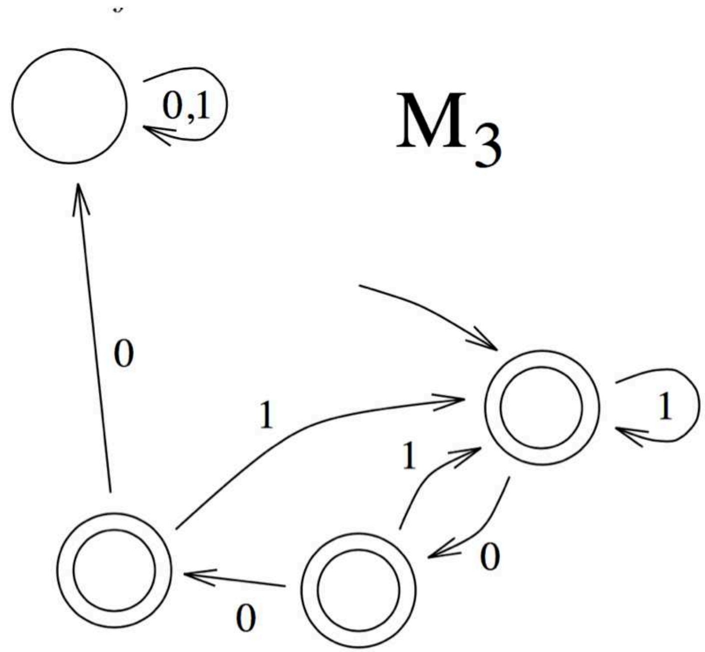
Task: Formal Language Recognition

Accept	Reject
1	0 0 0
0	1 1 0 0 0
1 0	0 0 0 1
0 1	0 0 0 0 0 0 0 0 0
0 0	1 1 1 1 1 0 0 0 0 1 1
1 0 0 1 0 0	1 1 0 1 0 1 0 0 0 0 0 1 0 1 1 1
0 0 1 1 1 1 1 1 0 1 0 0	1 0 1 0 0 1 0 0 0 1
0 1 0 0 1 0 0 1 0 0	0 0 0 0
1 1 1 0 0	0 0 0 0 0
0 0 1 0	

Scan a sequence of characters one at a time,
then classify the sequence as Accept or Reject.

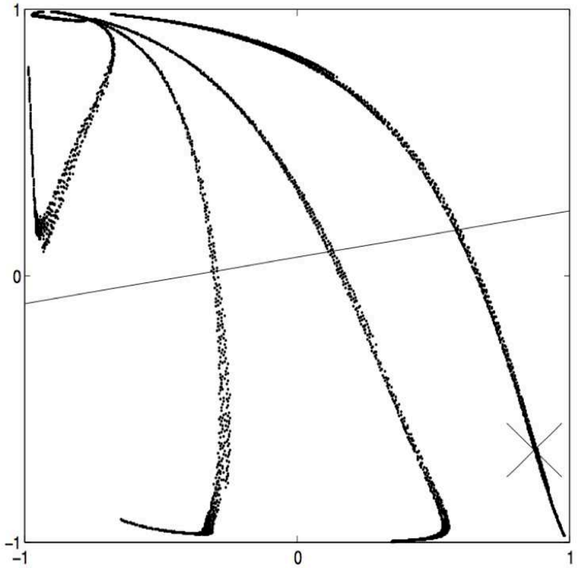
Dynamical Recognizers

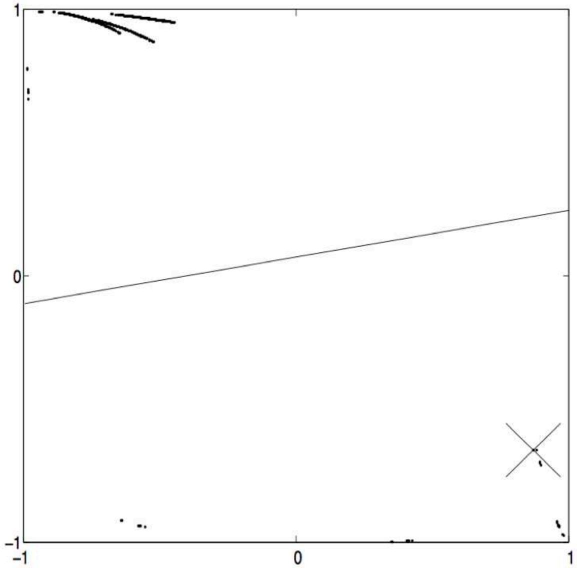




- trained network emulates the behaviour of Finite State Automaton
- training set must include short, medium and long examples

Phase Transition





$$W_0 = \begin{bmatrix} -0.567 & 1.761 & 0.815 \\ -0.219 & -2.591 & 0.446 \end{bmatrix} \qquad W_0 = \begin{bmatrix} -0.567 & 1.763 & 0.816 \\ -0.219 & -2.593 & 0.446 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0.752 & 0.548 & -1.071 \\ 0.074 & -0.813 & 1.502 \end{bmatrix} \qquad W_1 = \begin{bmatrix} 0.751 & 0.549 & -1.073 \\ 0.075 & -0.813 & 1.502 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.069 & 0.172 & -0.985 \end{bmatrix} \qquad P = \begin{bmatrix} 0.069 & 0.173 & -0.985 \end{bmatrix}$$

Chomsky Hierarchy

Language	Machine	Example
Regular	Finite State Automaton	a^n (n odd)
Context Free	Push Down Automaton	$a^n b^n$
Context Sensitive	Linear Bounded Automaton	$a^n b^n c^n$
Recursively Enumerable	Turing Machine	true QBF

Task: Formal Language Prediction

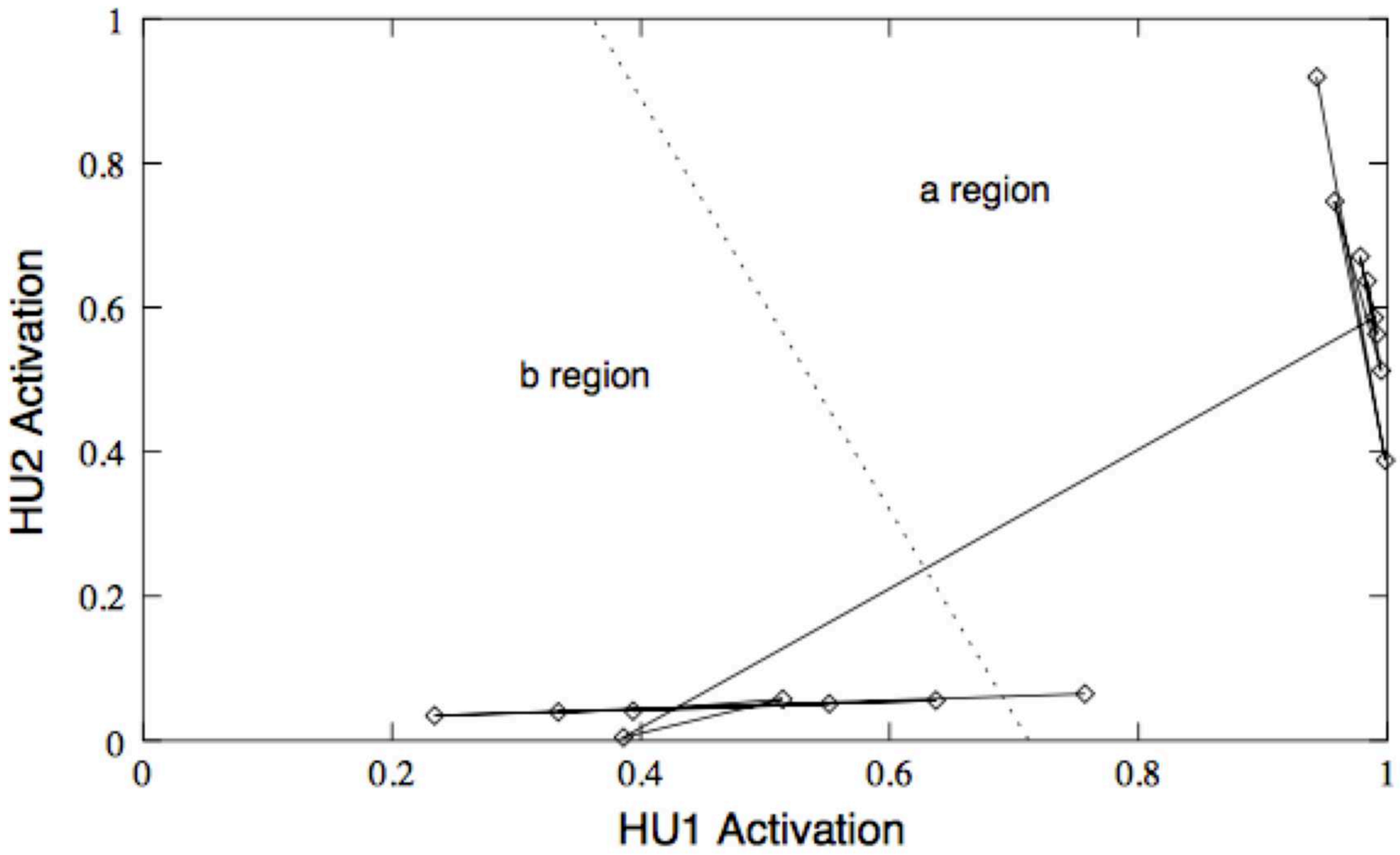
$abaabbabaaabbbaaaabbbbabaabbaaaaabbbbb\ldots$

Scan a sequence of characters one at a time, and try at each step to predict the next character in the sequence.

In some cases, the prediction is probabilistic.

For the $a^n b^n$ task, the first b is not predictable, but subsequent b's and the initial a in the next subsequence are predictable.

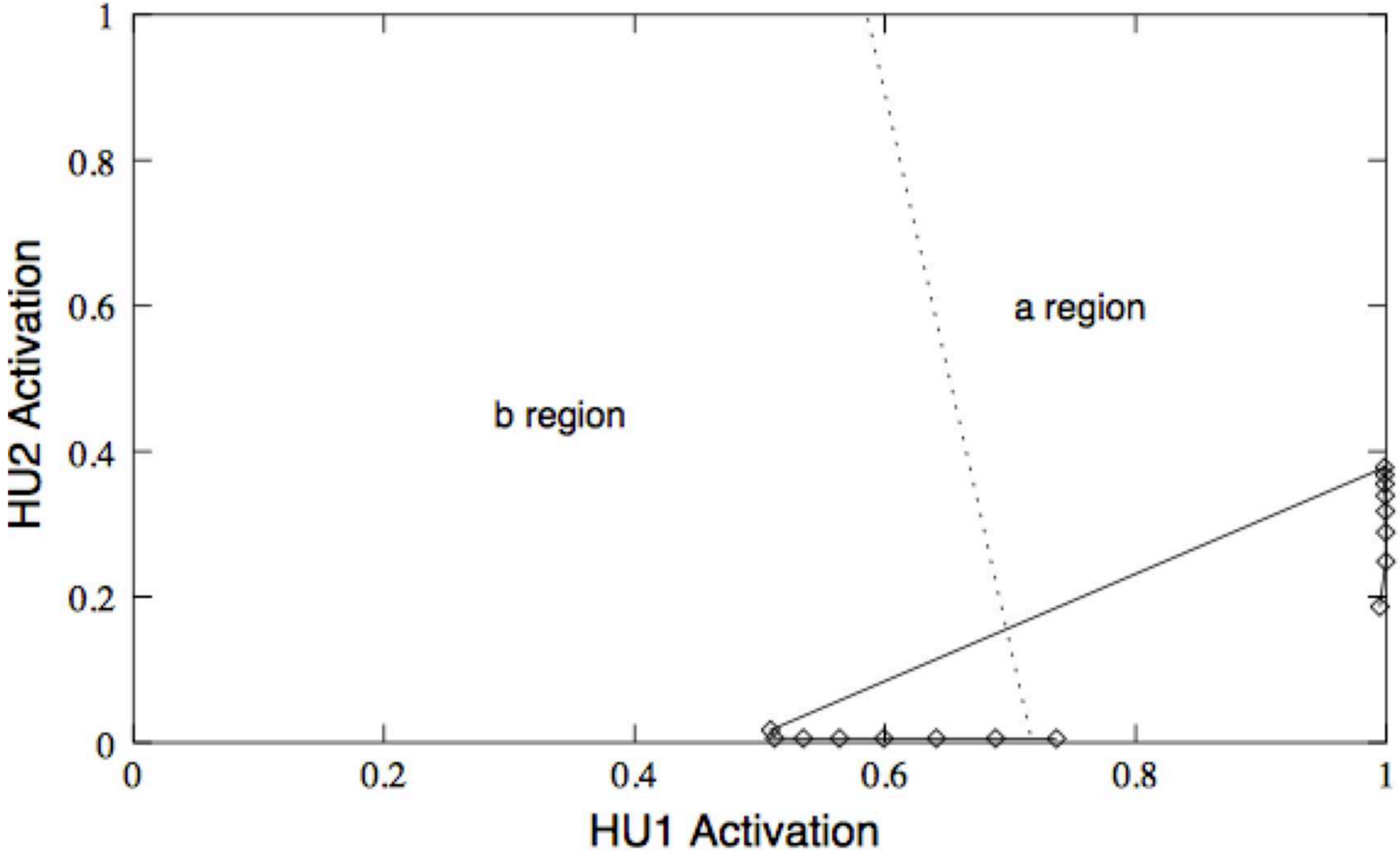
Oscillating Solution for $a^n b^n$



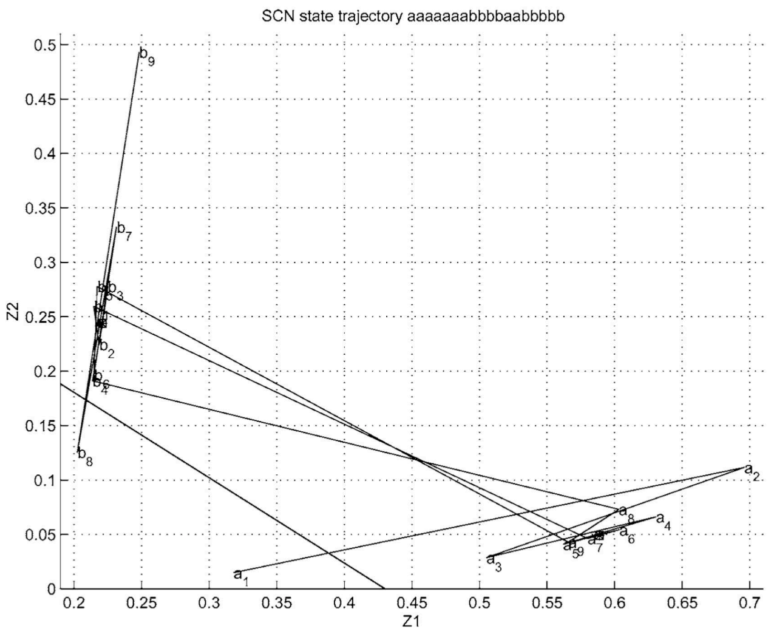
Learning to Predict $a^n b^n$

- the network does not implement a Finite State Automaton but instead uses two fixed points in activation space – one attracting, the other repelling (Wiles & Elman, 1995)
- networks trained only up to $a^{10}b^{10}$ could generalize up to $a^{12}b^{12}$
- training the weights by evolution is more stable than by backpropagation
- networks trained by evolution were sometimes monotonic rather than oscillating

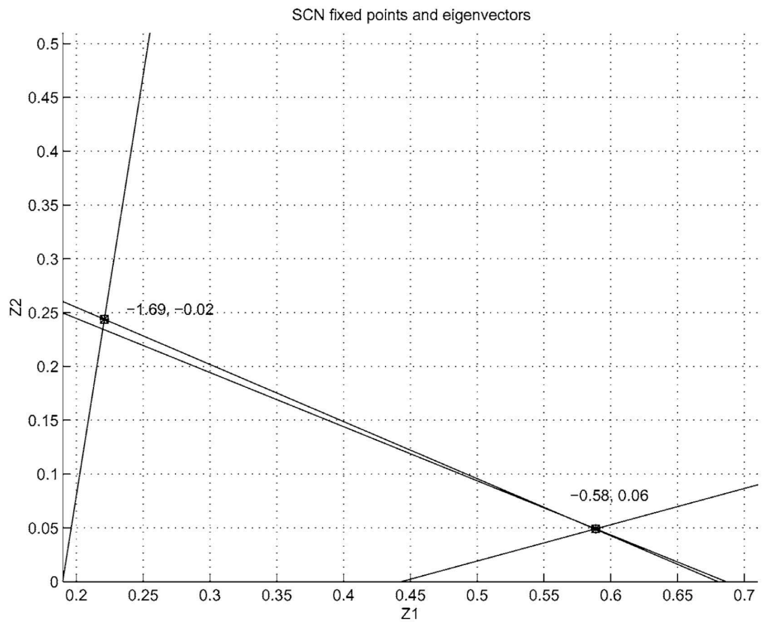
Monotonic Solution for $a^n b^n$



Hidden Unit Analysis for $a^n b^n$

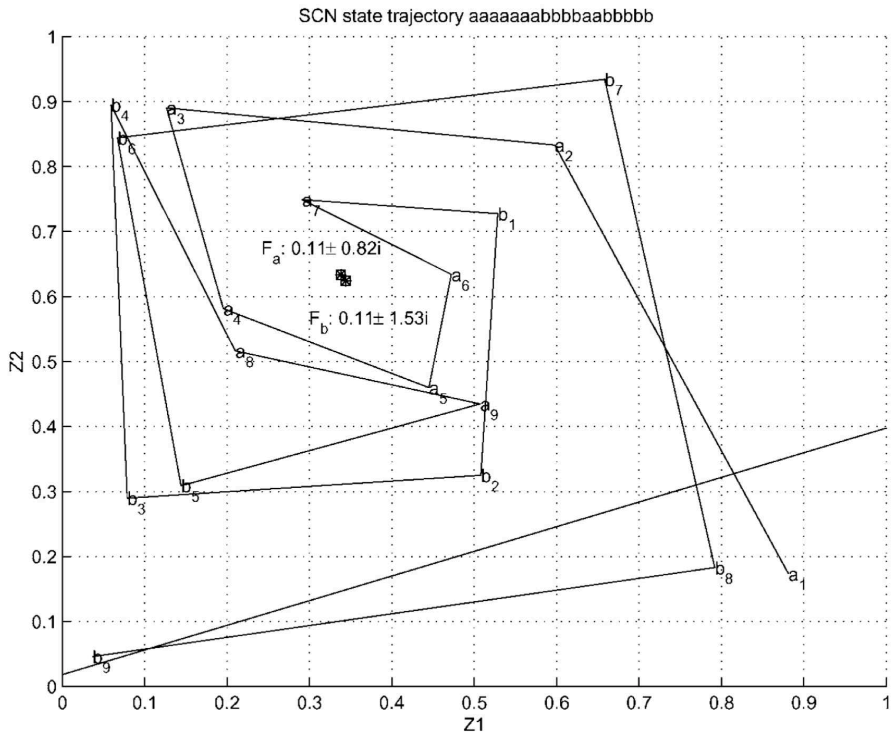


hidden unit trajectory



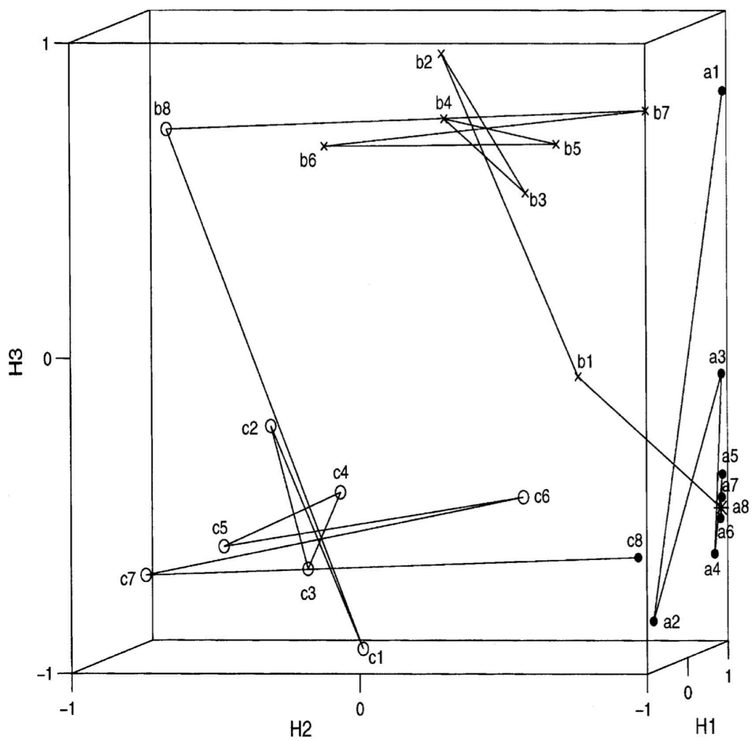
fixed points and eigenvectors

Counting by Spiralling



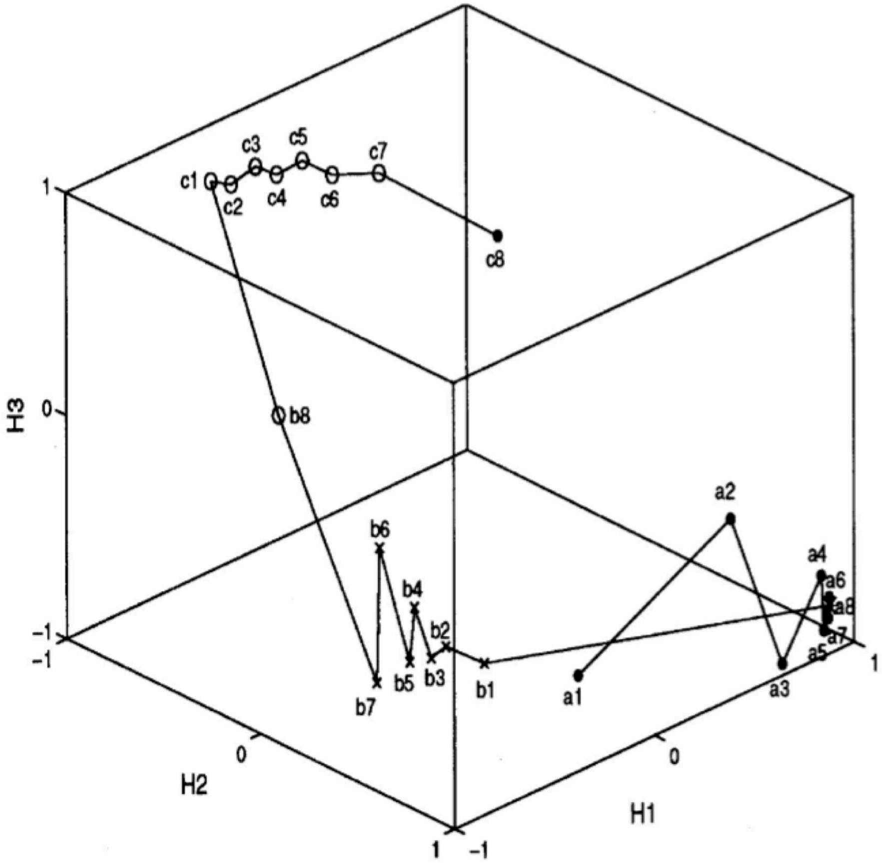
- for this task, sequence is accepted if the number of *a*’s and *b*’s are equal
- network counts up by spiralling inwards, down by spiralling outwards

Hidden Unit Dynamics for $a^n b^n c^n$

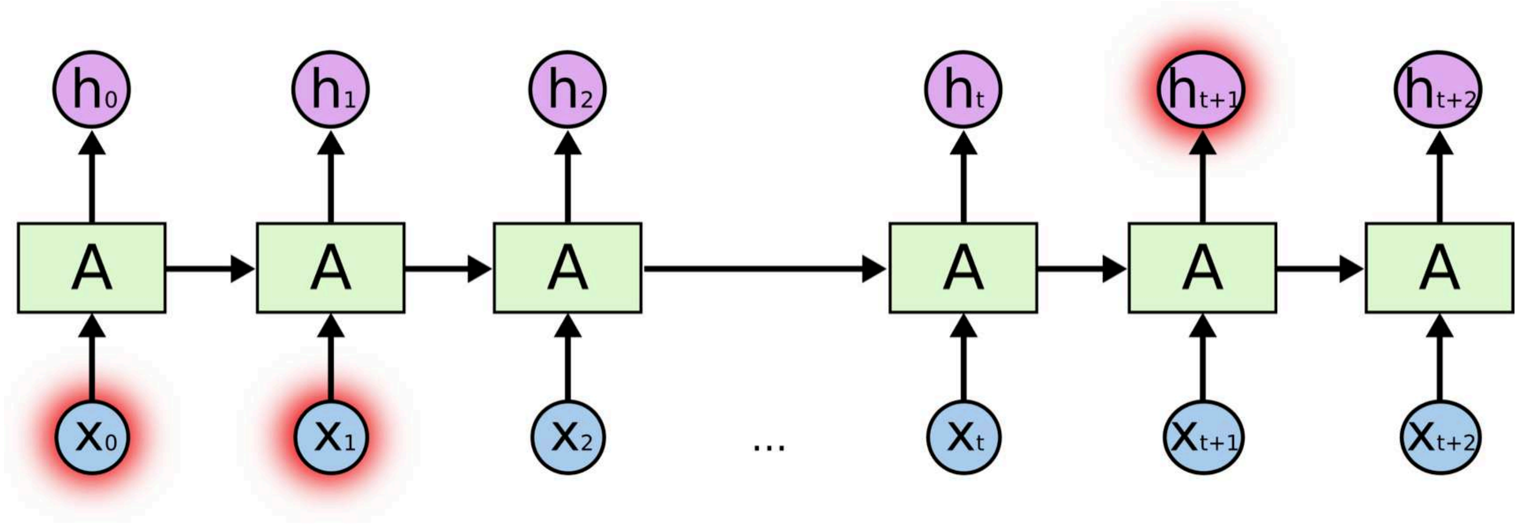


- SRN with 3 hidden units can learn to predict $a^n b^n c^n$ by counting up and down simultaneously in different directions, thus producing a star shape.

Partly Monotonic Solution for $a^n b^n c^n$



Long Range Dependencies



- Simple Recurrent Networks (SRNs) can learn medium-range dependencies but have difficulty learning long range dependencies
- Long Short Term Memory (LSTM) and Gated Recurrent Units (GRU) can learn long range dependencies better than SRN

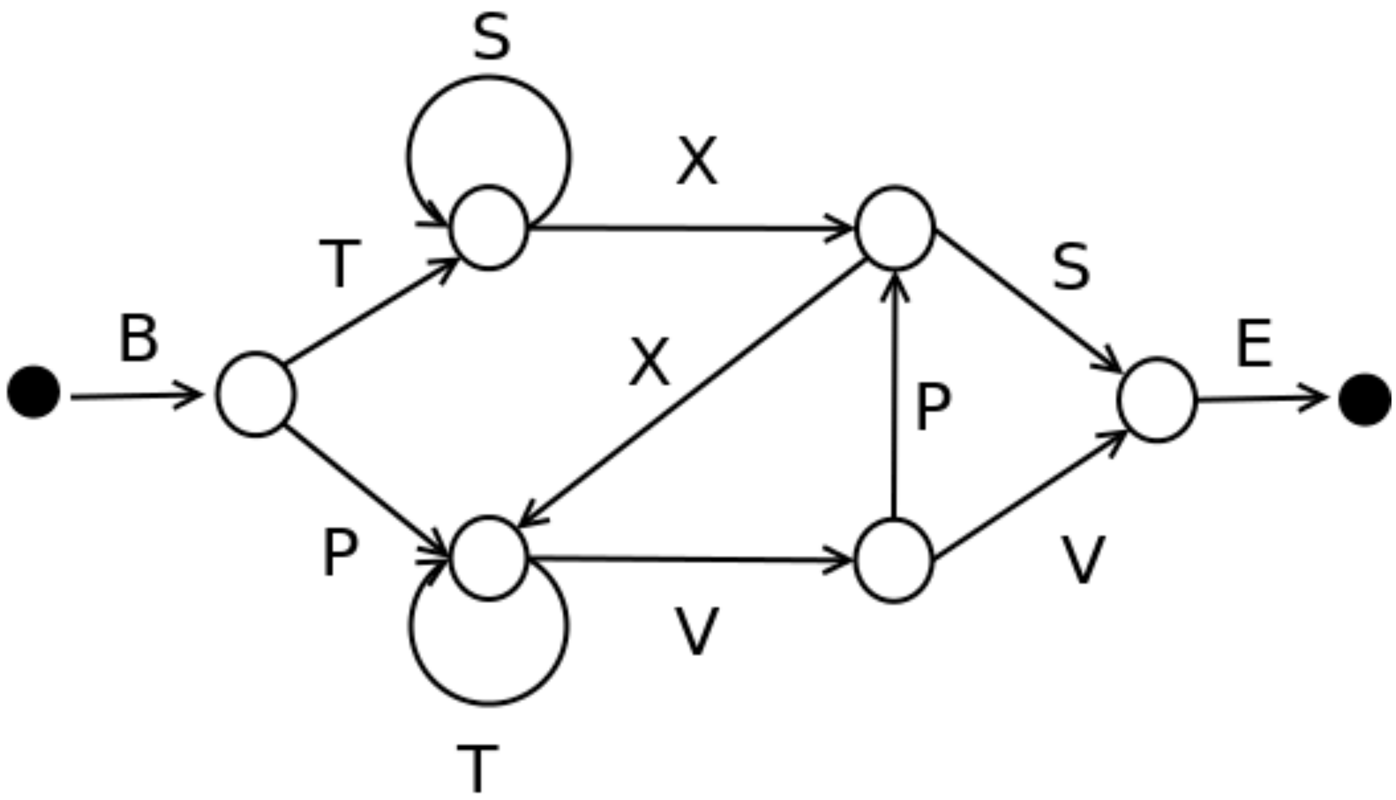
Long Short Term Memory

Two excellent Web resources for LSTM:

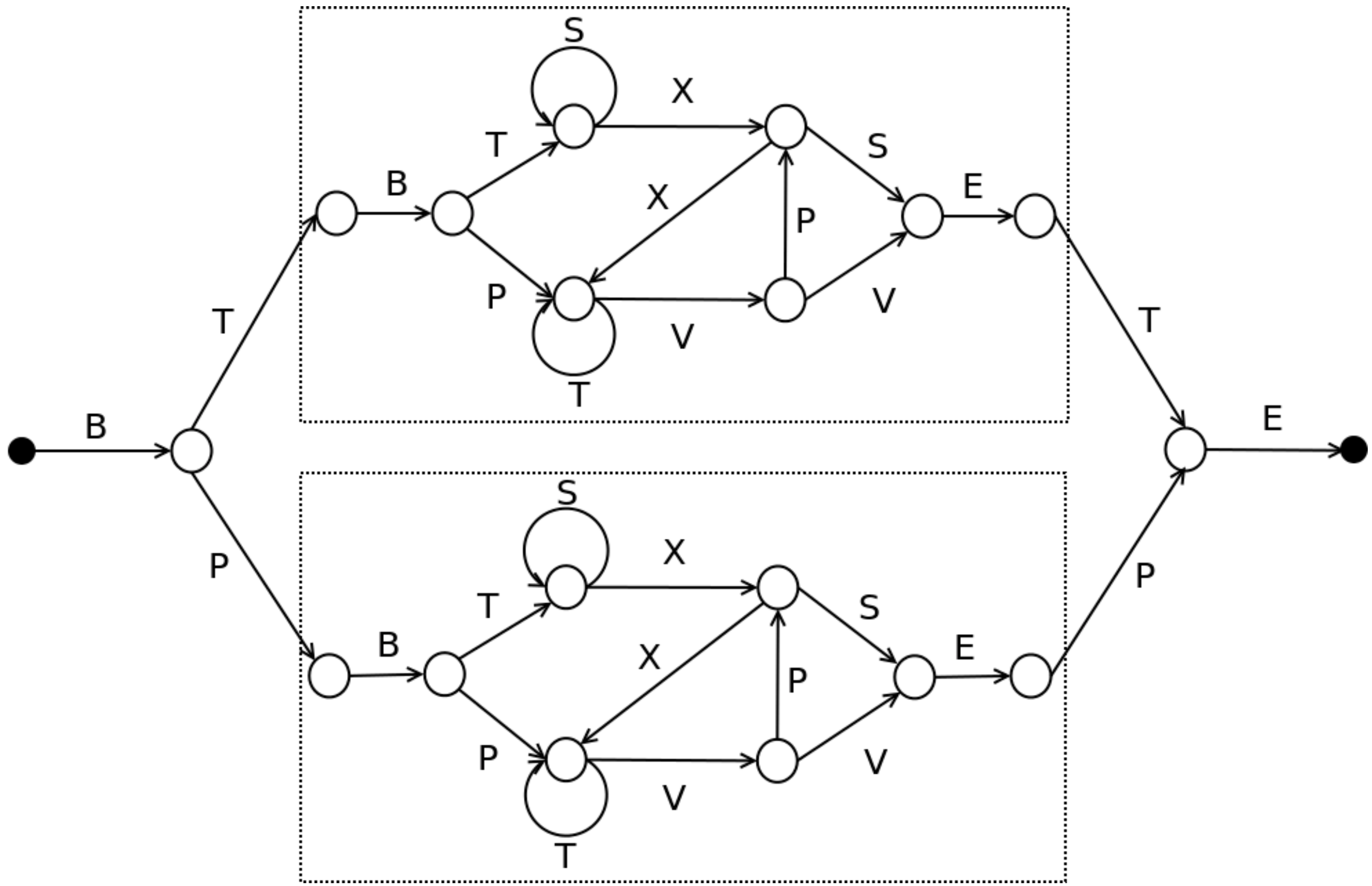
`http://colah.github.io/posts/2015-08-Understanding-LSTMs/`

`christianherta.de/lehre/dataScience/machineLearning/neuralNetworks/LSTM.php`

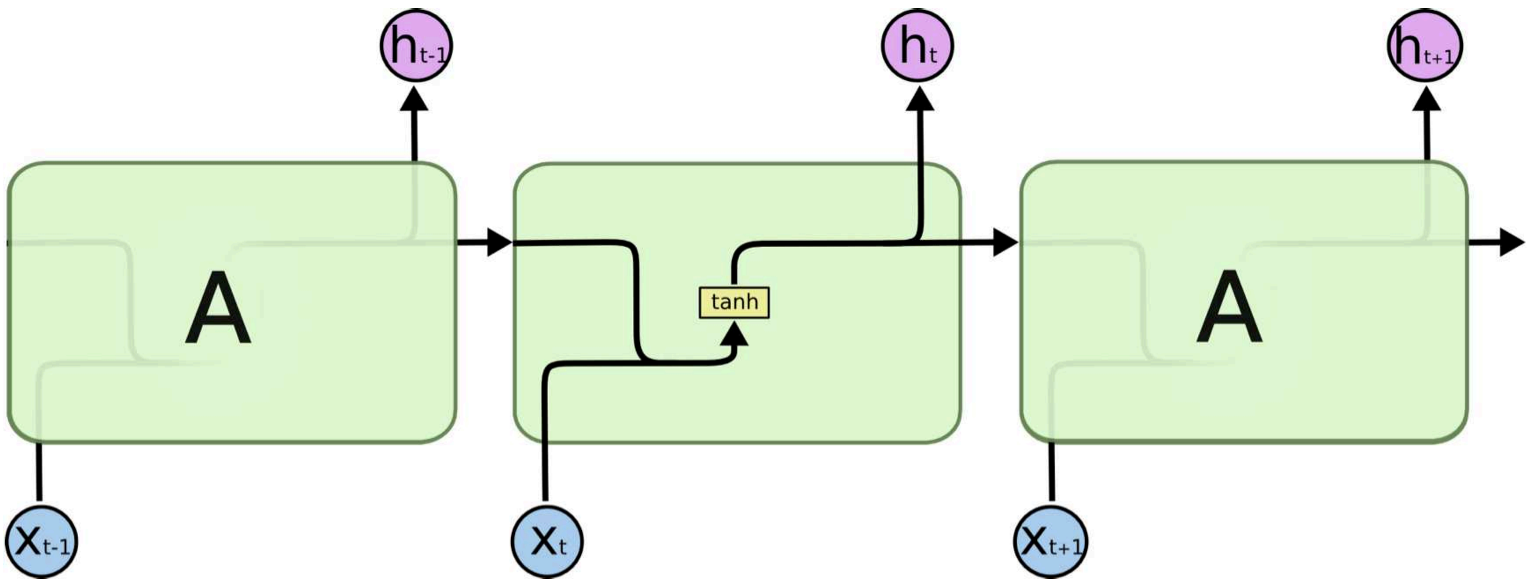
Reber Grammar



Embedded Reber Grammar



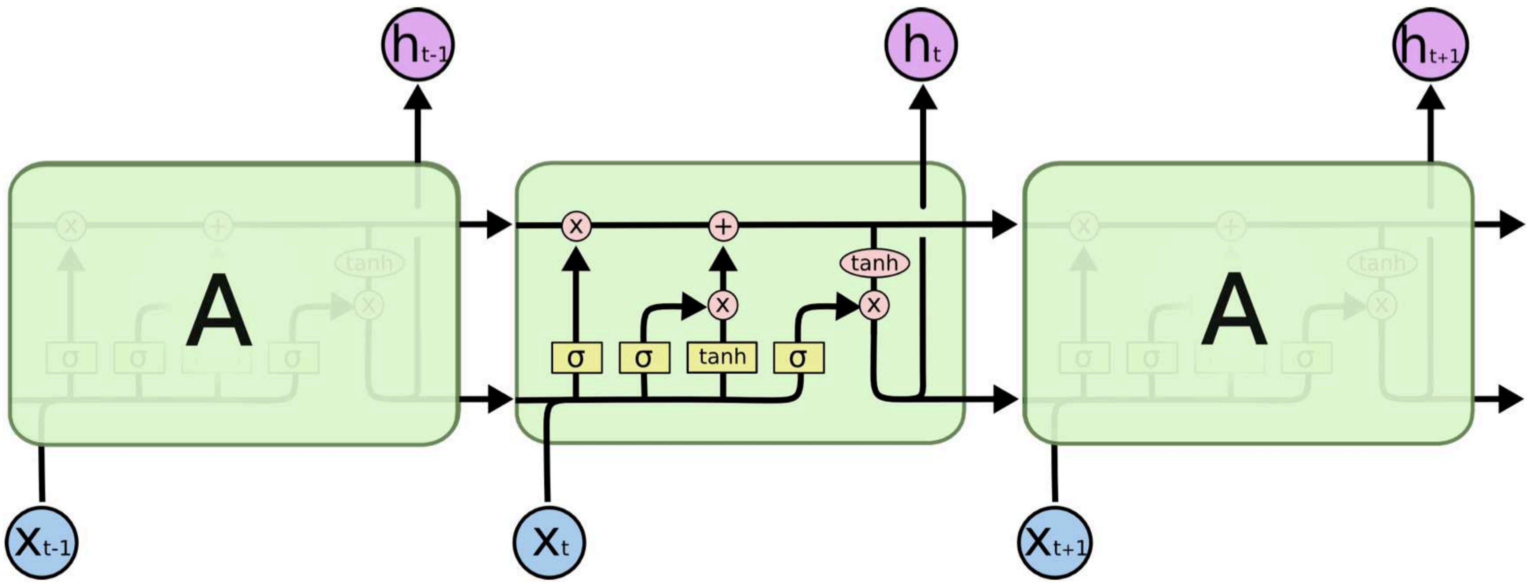
Simple Recurrent Network



SRN – context layer is combined directly with the input to produce the next hidden layer.

SRN can learn Reber Grammar, but not Embedded Reber Grammar.

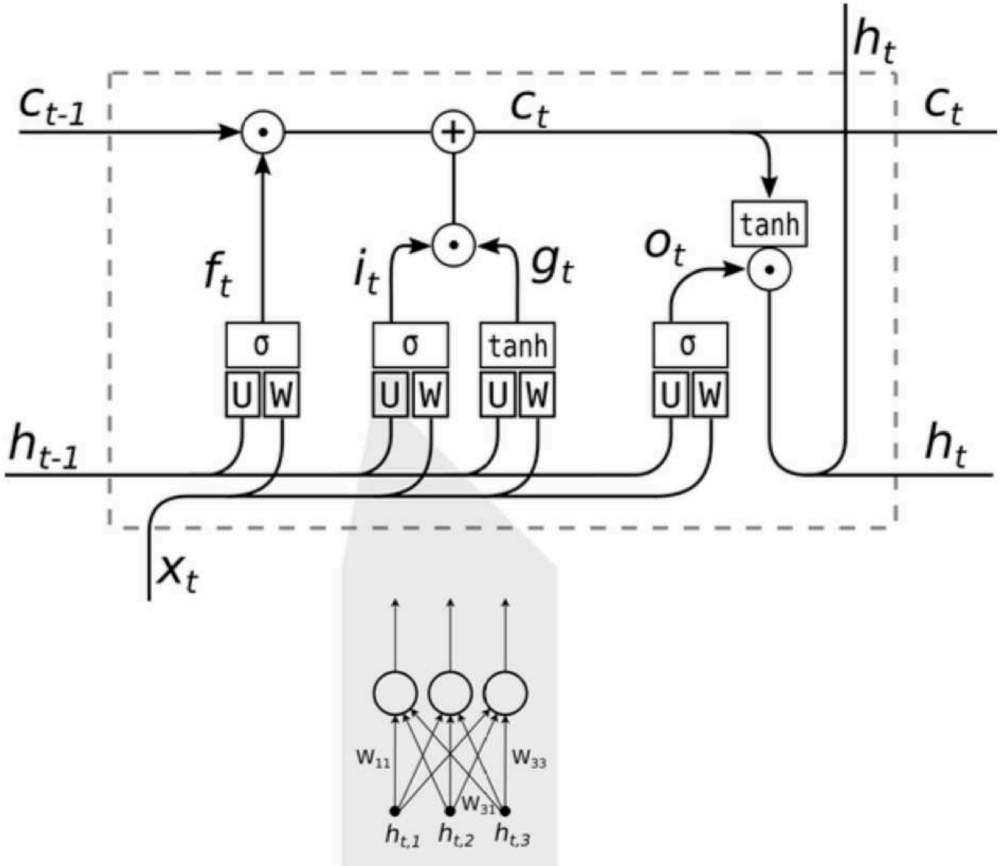
Long Short Term Memory



LSTM – context layer is modulated by three gating mechanisms: forget gate, input gate and output gate.

`http://colah.github.io/posts/2015-08-Understanding-LSTMs/`

Long Short Term Memory



Gates:

$$\mathbf{f}_t = \sigma\left(W_f \mathbf{x}_t + U_f \mathbf{h}_{t-1} + \mathbf{b}_f\right)$$
$$\mathbf{i}_t = \sigma\left(W_i \mathbf{x}_t + U_i \mathbf{h}_{t-1} + \mathbf{b}_i\right)$$
$$\mathbf{g}_t = \tanh\left(W_g \mathbf{x}_t + U_g \mathbf{h}_{t-1} + \mathbf{b}_g\right)$$
$$\mathbf{o}_t = \sigma\left(W_o \mathbf{x}_t + U_o \mathbf{h}_{t-1} + \mathbf{b}_o\right)$$

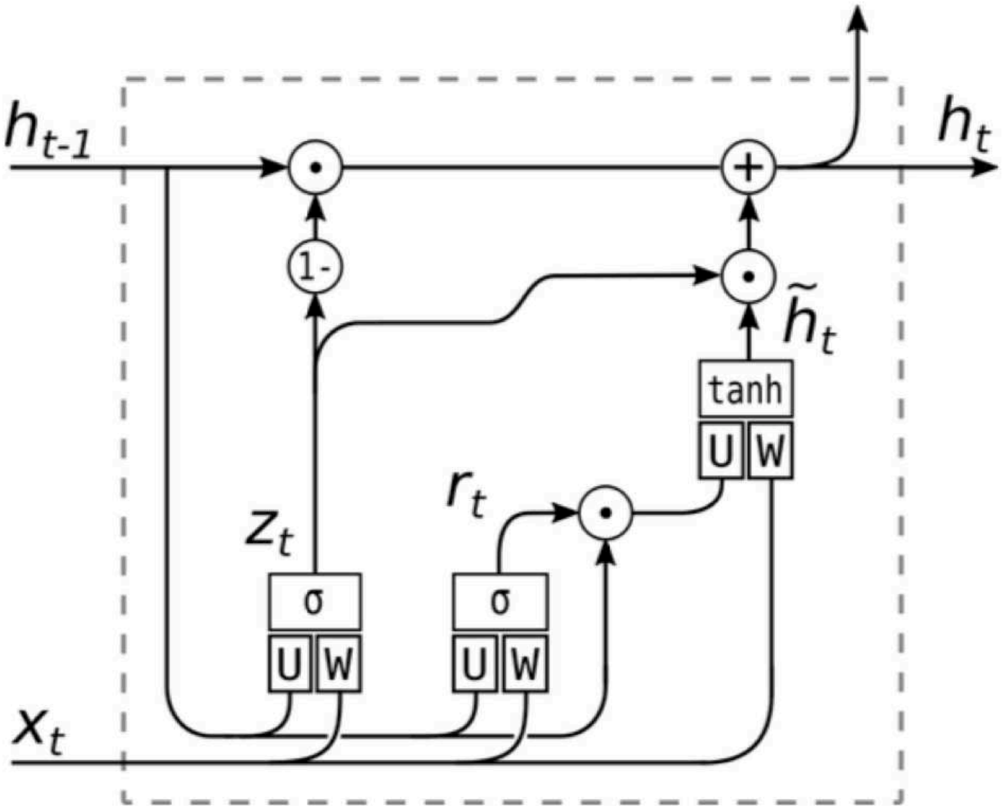
State:

$$\mathbf{c}_t = \mathbf{c}_{t-1} \odot \mathbf{f}_t + \mathbf{i}_t \odot \mathbf{g}_t$$

Output:

$$\mathbf{h}_t = \tanh \mathbf{c}_t \odot \mathbf{o}_t$$

Gated Recurrent Unit



Gates:

$$\mathbf{z}_t = \sigma(W_z\mathbf{x}_t + U_z\mathbf{h}_{t-1} + \mathbf{b}_z)$$

$$\mathbf{r}_t = \sigma(W_r\mathbf{x}_t + U_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$

Candidate Activation:

$$\tilde{\mathbf{h}}_t = \tanh\left(W\mathbf{x}_t + U(\mathbf{r}_t \odot \mathbf{h}_{t-1}) + \mathbf{b}_h\right)$$

Output:

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t$$

GRU is similar to LSTM but has only two gates instead of three.

Google Neural Machine Translation