Exercise sheet 2

COMP6741: Parameterized and Exact Computation

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Semester 2, 2017

Exercise 1. Prove the following generalization of Lemma 3 [Lawler '76]: For any graph G on n vertices, if G has a k-coloring, then G has a k-coloring where one color class is a maximal independent set in G of size at least n/k.

Exercise 2. In the MEETING MOST DEADLINES problem, we are given n tasks $t_1, \ldots, t_n$, and each task t_i has a length ℓ_i, a due date d_i, and a penalty p_i which applies when the due date of task t_i is not met. The problem asks to assign a start date $s_i \geq 0$ to each task t_i so that the executions of no two tasks overlap, and the sum of the penalties of those tasks that are not finished by the due date is minimized.

MEETING MOST DEADLINES

Input:	A set $T = \{t_1, \ldots, t_n\}$ of n tasks, where each task t_i is a triple (ℓ_i, d_i, p_i) of three non-negative integers.
Output:	A schedule, assigning a start date $s_i \in \mathbb{N}_0$ to each task $t_i \in T$ s.t. $$\sum_{i \in \{1,\ldots,n\} \,:\, s_i + \ell_i > d_i} p_i$$ is minimized, subject to the constraint that for every $i, j \in \{1, \ldots, n\}$ with $i \neq j$ we have that $s_i \notin \{s_j, s_j + 1, \ldots, s_j + \ell_j - 1\}$.

(a) Show that the MEETING MOST DEADLINES problem can be solved in $O^*(n!)$ time by reformulating it as a permutation problem.

(b) Design an algorithm solving the MEETING MOST DEADLINES problem in $O^*(2^n)$ time.