COMP4418: Knowledge Representation—Solutions to Exercise Set 2
First-Order Logic

1. (i) All birds fly
   (If an object x is a bird, then it flies.)

   (ii) Everyone has a mother

   (iii) There is someone who is everyone's mother

2. (i) $\forall x.(cat(x) \rightarrow mammal(x))$

   (ii) $\neg \exists x.(cat(x) \land reptile(x))$
   or, equivalently, $\forall x.(cat(x) \rightarrow \neg reptile(x))$

   (iii) $\forall x.\exists y.(computer\_scientist(x) \rightarrow likes(x, y))$

3. (i) CNF($\forall x.(bird(x) \rightarrow flies(x))$)
   $\equiv \forall x.(\neg bird(x) \lor flies(x))$ (Remove $\rightarrow$)
   $\equiv \neg bird(x) \lor flies(x)$ (Drop $\forall$))

   (ii) CNF($\exists x.\forall y.\forall z.(person(x) \land ((likes(x, y) \land yneqz) \rightarrow \neg likes(x, z))))$
   $\equiv \exists x.\forall y.\forall z.(person(x) \land (\neg(likes(x, y) \land y \neq z) \lor \neg likes(x, z)))$ (Remove $\rightarrow$)
   $\equiv \exists x.\forall y.\forall z.(person(x) \land (\neg likes(x, y) \lor y = z \lor \neg likes(x, z)))$ (De Morgan)
   $\equiv \forall y.\forall z.(person(x) \land (\neg likes(c, y) \lor y = z \lor \neg likes(c, z)))$ (Skolemisation—c is a constant(
   $\equiv person(c) \land (\neg likes(c, y) \lor y = z \lor \neg likes(c, z))$ (Drop $\forall$)

4. (i) CNF($\forall x.(P(x) \rightarrow Q(x))$)
   $\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\rightarrow$)
   $\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)

   CNF($\neg \forall x.(\neg Q(y) \rightarrow \neg P(y))$)
   $\equiv \neg \forall x.(\neg\neg Q(y) \lor \neg P(y))$ (Remove $\rightarrow$)
   $\equiv \exists x.\neg(\neg\neg Q(y) \lor \neg P(y))$ (De Morgan)
   $\equiv \exists x.\neg(Q(y) \lor \neg P(y))$ (Double Negation)
   $\equiv \exists x.(\neg Q(y) \land \neg\neg P(y))$ (De Morgan)
   $\equiv \exists x.(\neg Q(y) \land P(y))$ (Double Negation)
   $\equiv \neg Q(c) \land P(c)$ (Skolemisation)

   Proof:

1.	$\neg P(x) \lor Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \lor Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\Box$	3, 5 Resloution

(ii) (Works exactly as in (i).)

CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)

CNF($\neg\forall x.(\neg Q(x) \to \neg P(x))$)
$\equiv \neg\forall x.(\neg\neg Q(x) \lor \neg P(x))$ (Remove $\to$)
$\equiv \neg\forall x.(Q(x) \lor \neg P(x))$ (Double Negation)
$\equiv \exists x.\neg(Q(x) \lor \neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \land \neg\neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \land P(x))$ (Double Negation)
$\equiv \neg Q(c) \land \neg P(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \lor Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \lor Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\square$	3, 5 Resloution

(iii) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)

CNF($P(a)$)
$\equiv P(a)$

CNF($\neg Q(a)$)
$\equiv \neg Q(a)$

Proof:

1.	$\neg P(x) \lor Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(a)$	(Negated Conclusion)
4.	$\neg P(a) \lor Q(a)$	(1. $\{x/a\}$)
5.	$\neg Q(a)$	2, 4 Resolution
6.	$\square$	3, 5 Resolution

(iv) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)

CNF($\exists x.P(x)$)
$\equiv P(a)$ (Skolemisation)

CNF($\neg\exists x.Q(x)$)
$\equiv \forall x.\neg Q(x)$ (De Morgan)

$\equiv \neg Q(x)$ (Drop $\forall$)

Proof:

1.	$\neg P(x) \lor Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(y)$	(Negated Conclusion)
4.	$\neg P(a) \lor Q(a)$	(1. $\{x/a\}$)
5.	$Q(a)$	2, 4 Resolution
6.	$\neg Q(a)$ (3. $\{y/a\}$)	
7.	$\square$	5, 6 Resolution

(v) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)

CNF($\forall x.(Q(x) \to R(x))$)
$\equiv \forall x.(\neg Q(x) \lor R(x))$ (Remove $\to$)
$\equiv \neg Q(x) \lor R(x)$ (Drop $\forall$)

CNF($\neg \forall x.(P(x) \to R(x))$)
$\equiv \neg \forall x.(\neg P(x) \lor R(x))$ (Remove $\to$)
$\equiv \exists x.(\neg(\neg P(x) \lor R(x))$ (De Morgan)
$\equiv \exists x.(\neg\neg P(x) \land \neg R(x))$ (De Morgan)
$\equiv \exists x.(P(x) \land \neg R(x))$ (Double Negation)
$\equiv P(c) \land \neg R(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \lor Q(x)$	(Hypothesis)
2.	$\neg Q(y) \lor R(y)$	(Hypothesis)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg R(c)$	(Negated Conclusion)
5.	$\neg P(c) \lor Q(c)$	(1. $\{x/c\}$)
6.	$\neg Q(c) \lor R(c)$	(2. $\{y/c\}$)
7.	$\neg P(c) \lor R(c)$	5, 6 Resolution
8.	$R(c)$	3, 7 Resolution
9.	$\square$	4, 8 Resolution

5. (i) (A) $\exists x.\forall y.(cs(x) \land likes(x, y))$
(B) $os(Linux)$
(C) $\exists z.likes(z, Linux)$

(ii) (A) $cs(c) \land likes(c, y)$ (Skolemisation and Drop $\forall$)
(B) $os(Linux)$
(C) $\neg likes(z, Linux)$ (De Morgan and Drop $\forall$)

(iii)

1.	$cs(c)$	(Hypothesis A)
2.	$likes(c, y)$	(Hypothesis A)
3.	$os(Linux)$	(Hypothesis B)
4.	$\neg likes(z, Linux)$	(Negated Consequence)
5.	$likes(c, Linux)$	(2. $\{y/Linux\}$)
6.	$\neg likes(c, Linux)$	(4. $\{z/c\}$)
7.	$\square$	5, 6 Resolution

(iv) Yes. A, B, $\neg C$ in (ii) are Horn clauses so there must be an SLD resolution of the empty clause if there is a resolution of the empty clause and there is as we have seen in (ii). In fact, the resolution in (iii) is an SLD resolution of the empty clause.

(v) $A, B \vdash C$