COMP4418: Knowledge Representation—Solutions to Exercise Set 2
First-Order Logic

1. (i) All birds fly
   (If an object x is a bird, then it flies.)

   (ii) Everyone has a mother

   (iii) There is someone who is everyone's mother

2. (i) $\forall x(Cat(x) \rightarrow Mammal(x))$

   (ii) $\neg\exists x(Cat(x) \wedge Reptile(x))$
   or, equivalently, $\forall x(Cat(x) \rightarrow \neg Reptile(x))$

   (iii) $\forall x\exists y(ComputerScientist(x) \rightarrow OS(y) \wedge Likes(x, y))$

3. (i) CNF($\forall x(Bird(x) \rightarrow Flies(x))$)
   $\equiv \forall x(\neg Bird(x) \vee Flies(x))$ (Remove $\rightarrow$)
   $\equiv \neg Bird(x) \vee Flies(x)$ (Drop $\forall$))

   (ii) CNF($\exists x\forall y\forall z(Person(x) \wedge ((Likes(x, y) \wedge y \neq z) \rightarrow \neg Likes(x, z)))$)
   $\equiv \exists x\forall y\forall z(Person(x) \wedge (\neg(Likes(x, y) \wedge y \neq z) \vee \neg Likes(x, z)))$ (Remove $\rightarrow$)
   $\equiv \exists x\forall y\forall z(Person(x) \wedge (\neg Likes(x, y) \vee y = z \vee \neg Likes(x, z)))$ (De Morgan)
   $\equiv \forall y\forall z(Person(x) \wedge (\neg Likes(c, y) \vee y = z \vee \neg Likes(c, z)))$ (Skolemisation—c is a constant)
   $\equiv Person(c) \wedge (\neg Likes(c, y) \vee y = z \vee \neg Likes(c, z))$ (Drop $\forall$)

4. (i) CNF($\forall x(P(x) \rightarrow Q(x))$)
   $\equiv \forall x(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
   $\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

   CNF($\neg\forall x(\neg Q(y) \rightarrow \neg P(y))$)
   $\equiv \neg\forall x(\neg\neg Q(y) \vee \neg P(y))$ (Remove $\rightarrow$)
   $\equiv \exists x\neg(\neg\neg Q(y) \vee \neg P(y))$ (De Morgan)
   $\equiv \exists x\neg(Q(y) \vee \neg P(y))$ (Double Negation)
   $\equiv \exists x(\neg Q(y) \wedge \neg\neg P(y))$ (De Morgan)
   $\equiv \exists x(\neg Q(y) \wedge P(y))$ (Double Negation)
   $\equiv \neg Q(c) \wedge P(c)$ (Skolemisation)

   Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1 $\{x/c\}$)
5.	$\neg P(c)$	(2, 4 Resolution)
6.	$\Box$	(3, 5 Resloution)

   (ii) (Works exactly as in (i).)

   CNF($\forall x(P(x) \rightarrow Q(x))$)
   $\equiv \forall x(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
   $\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

   CNF($\neg\forall x(\neg Q(x) \rightarrow \neg P(x))$)
   $\equiv \neg\forall x(\neg\neg Q(x) \vee \neg P(x))$ (Remove $\rightarrow$)
   $\equiv \neg\forall x(Q(x) \vee \neg P(x))$ (Double Negation)

$\equiv \exists x \neg(Q(x) \vee \neg P(x))$ (De Morgan)
$\equiv \exists x(\neg Q(x) \wedge \neg\neg P(x))$ (De Morgan)
$\equiv \exists x(\neg Q(x) \wedge P(x))$ (Double Negation)
$\equiv \neg Q(c) \wedge \neg P(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1 $\{x/c\}$)
5.	$\neg P(c)$	(2, 4 Resolution)
6.	$\square$	(3, 5 Resloution)

(iii) CNF($\forall x(P(x) \to Q(x))$)
$\equiv \forall x(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($P(a)$)
$\equiv P(a)$

CNF($\neg Q(a)$)
$\equiv \neg Q(a)$

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(a)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1 $\{x/a\}$)
5.	$\neg Q(a)$	(2, 4 Resolution)
6.	$\square$	(3, 5 Resolution)

(iv) CNF($\forall x(P(x) \to Q(x))$)
$\equiv \forall x(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\exists x P(x)$)
$\equiv P(a)$ (Skolemisation)

CNF($\neg \exists x Q(x)$)
$\equiv \forall x \neg Q(x)$ (De Morgan)
$\equiv \neg Q(x)$ (Drop $\forall$)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(y)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1 $\{x/a\}$)
5.	$Q(a)$	(2, 4 Resolution)
6.	$\neg Q(a)$	(3 $\{y/a\}$)
7.	$\square$	(5, 6 Resolution)

(v) CNF($\forall x(P(x) \to Q(x))$)
$\equiv \forall x(\neg P(x) \vee Q(x))$ (Remove $\to$)

$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\forall x(Q(x) \to R(x))$)
$\equiv \forall x(\neg Q(x) \vee R(x))$ (Remove $\to$)
$\equiv \neg Q(x) \vee R(x)$ (Drop $\forall$)

CNF($\neg\forall x(P(x) \to R(x))$)
$\equiv \neg\forall x(\neg P(x) \vee R(x))$ (Remove $\to$)
$\equiv \exists x \neg(\neg P(x) \vee R(x))$ (De Morgan)
$\equiv \exists x(\neg\neg P(x) \wedge \neg R(x))$ (De Morgan)
$\equiv \exists x(P(x) \wedge \neg R(x))$ (Double Negation)
$\equiv P(c) \wedge \neg R(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(y) \vee R(y)$	(Hypothesis)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg R(c)$	(Negated Conclusion)
5.	$\neg P(c) \vee Q(c)$	(1 $\{x/c\}$)
6.	$\neg Q(c) \vee R(c)$	(2 $\{y/c\}$)
7.	$\neg P(c) \vee R(c)$	5, 6 Resolution
8.	$R(c)$	3, 7 Resolution
9.	$\square$	4, 8 Resolution

5. (i) (A) $\exists x \forall y(ComputerScientist(x) \wedge (OS(y) \to Likes(x, y)))$
   (B) $OS(Linux)$
   (C) $\exists z Likes(z, Linux)$

(ii) (A) $ComputerScientist(c) \wedge (\neg OS(y) \vee Likes(c, y))$ (Remove $\to$, Skolemisation, and Drop $\forall$)
   (B) $OS(Linux)$
   (C) $\neg Likes(z, Linux)$ (De Morgan and Drop $\forall$)

(iii)

1.	$ComputerScientist(c)$	(Hypothesis A)
2.	$\neg OS(y) \vee Likes(c, y)$	(Hypothesis A)
3.	$OS(Linux)$	(Hypothesis B)
4.	$\neg Likes(z, Linux)$	(Negated Consequence)
5.	$\neg OS(Linux) \vee Likes(c, Linux)$	(2 $\{y/Linux\}$)
6.	$\neg Likes(c, Linux)$	(4 $\{z/c\}$)
7.	$Likes(c, Linux)$	(3, 5 Resolution)
8.	$\square$	(6, 7 Resolution)

(iv) Yes. A, B, $\neg C$ in (ii) are Horn clauses so there must be an SLD resolution of the empty clause if there is a resolution of the empty clause and there is as we have seen in (ii). In fact, the resolution in (iii) is an SLD resolution of the empty clause.

(v) $A, B \models C$