Family Name:

Other Names:

Signature:

Student Number:

This PAPER is NOT to be retained by the STUDENT

The University Of New South Wales

C0MP4418 Practice Exam (Not Marked!)

Knowledge Representation and Reasoning

October 2018

Time allowed: **2 Hours plus 10 Minutes reading time**
Total number of questions: **27**
Total number of marks: **100**

Questions in PART A, must be answered on the generalised answer sheet provided.
Questions in PART B, PART C and PART D must be answered in the answer book(s) provided.
You must hand in this entire exam paper and ALL your answer booklets. Otherwise you will get zero marks for the exam and a possible charge of academic misconduct.
Ensure that you fill in all of the details on the front of this pink paper, generalised answer sheet, and answer booklet(s) and then SIGN everything. This exam paper is printed single-sided so that you can use the reverse side of each page for working. You must hand this paper back with your generalised answer sheet and answer booklets at the conclusion of the exam.

Do not use red pen or pencil in the answer booklets for this exam.

No examination materials permitted.
Calculators may **not** be used.
Questions are **not** worth equal marks.
Answer **all** questions.

Part A: Multiple Choice Questions

NOTE: Answer the questions in this section on the generalised answer sheet provided.

Note that each question has *five* alternatives. Once you have chosen an alternative, fill in the multiple-choice answer sheet by giving the letter (in square brackets e.g., "[B]") which corresponds to that alternative. Also, be careful that you fill each answer in on the correct row on the multiple-choice sheet (i.e., the row corresponding to the question number).

Each question in this section is worth **2 marks**. There is a **penalty** of $-\frac{1}{2}$ **mark** for answering a question in this section incorrectly. There is no penalty for not answering a question. In other words, you get no marks for a question if you do not attempt it and you lose half a mark for getting a question wrong.

DO NOT answer these questions in an **answer booklet** or this **question paper**!

Question 1

Which of the following propositional formulas is a tautology?

[A] $p \rightarrow (q \rightarrow p)$.
[B] p.
[C] $p \vee p$
[D] $(p \rightarrow q) \wedge (q \rightarrow p)$.
[E] $\neg\neg p$.

A

- A tautology is a formula that will evaluate to true under every truth value assignment.
- First, eliminate the obvious candidates p, $p \vee p$, and $\neg\neg p$ are all false when p is false.
- Now, notice that $(p \rightarrow q) \wedge (q \rightarrow p)$ is the same as $(p \leftrightarrow q)$ which will be false whenever the values of p and q are different.
- Hence the only option left is $p \rightarrow (q \rightarrow p)$. Can confirm that this is a tautology by building a truth table and verifying that it is true for all rows.

Question 2

How many positive literals can appear in a definite clause?

[A] At most one.
[B] At least one.
[C] At least one but no more than three.
[D] Exactly one.
[E] Zero or more.

D

- Week 3 lecture: Horn logic, slide 17.
- A *Horn clause* has **at most** one positive literal.
- A *definite clause* is the special case of a Horn clause with **exactly one** positive literal.

Question 3

SLD resolution is most appropriate when the knowledge base consists entirely of?

[A] Arbitrary formulas.
[B] Arbitrary clauses.
[C] Horn clauses.
[D] Negative clauses.
[E] None of the above.

C

- Week 3 lecture: Horn logic, slide 24.
- SLD resolution is not *complete* in general clauses, but is complete for Horn clauses.

Question 4

Which of the following formal approaches to reasoning tries to capture commonsense reasoning?

[A] Default logic.
[B] First-order logic.
[C] Propositional logic.
[D] Resolution.
[E] None of the above.

A

- Week 4 lecture: Nonmonotonic reasoning, slide 4-6.
- First-order logic (FOL) and propositional logic (PL) are *monotonic* (i.e., the more facts we have the more conclusions we can draw). Resolution is an inference mechanism for FOL and PL.
- Default logic is a type of *non-monotonic* logic. That is, a logic that allows us to retract inferences when we received more information. So we initially reason that Tweety the bird can fly, but when we find out that Tweety is an emu then we conclude that it can't fly.

Question 5

In Prolog, rules correspond to which type of formulas? Give the most approporiate answer.

[A] Arbitrary formulas.
[B] Arbitrary clauses.
[C] Definite clauses.
[D] Facts.
[E] Horn clauses.

C

- All Prolog rules have a *head* (i.e., the positive literal in a definite clause).

Question 6

In first-order logic, how would you express that "something likes something"?

[A] $\exists x \exists y Likes(x, y)$
[B] $\exists x \forall y Likes(x, y)$
[C] $\forall x \exists y Likes(x, y)$
[D] $\forall x \forall y Likes(x, y)$
[E] None of the above.

A

- Week 2: First-order logic, slides 6-8.
- In English: [A] - something likes something, [B] - something likes everything, [C] - everything likes something, [D] - everything likes everything.

Question 7

Which of the following is not required to convert a formual into conjunctive normal form?

[A] Drop universal quantifiers.
[B] Eliminate implication.
[C] Resolve two clauses with complementary literals.
[D] Skolemisation.
[E] Standardise variables.

C

- Week 2: First-order logic, slide 11 - 13.
- Firstly [A],[B],[D],[E] are all part of the process outlined in slides 11,12.
- While [C] is about the resolution proof system, and not conversion.

Question 8

What is the idea behind Conflict-Driven Clause Learning?

[A] Add new literals that follow from the input CNF formula and the current partial interpretation.
[B] Add new clauses that follow from the input CNF formula and the current partial interpretation.
[C] Add new clauses to randomize the search tree.
[D] Add new clauses (in case of a conflict) that to avoid making similar assignments that lead to conflicts in future.
[E] Delete clauses that have low activity score.

D

- Week 7: Slide 30
- When a partial interpretation $\{x_1, \ldots, x_k\}$ leads to a conflict, clause learning attempts to analyse what the cause of this conflict is.
- The cause is a subset $\{x_{i_1}, \ldots, x_{i_\ell}\}$ of the partial interpretation. Every partial interpretation that contains these literals cannot be extended to become a satisfying interpretation.
- Thus we want to prevent the solver from adding $x_{i_1}, \ldots, x_{i_\ell}$ at any later point during the search. The conflict clause $\{\bar{x}_{i_1}, \ldots, \bar{x}_{i_\ell}\}$ does this: whenever $x_{i_1}, \ldots, x_{i_\ell}$ is added, the new conflict clause will cause a conflict.

Question 9

Which of the following statements about SAT and k-SAT is **false**?

[A] 4-SAT is not NP-complete.
[B] SAT is commonly believed to require exponential time.
[C] SAT is can be reduced to 3-SAT in polynomial time.
[D] 3-SAT and SAT are NP-complete.
[E] 2-SAT is in P and is hence known to be efficiently solvable.

A

- Week 6: Slides 13, 14, 17–19, 39
- 2-SAT is in P
- k-SAT is NP-complete for $k \geq 3$

Question 10

How many stable models can an ASP program have?

[A] Exactly zero.
[B] Exactly one.
[C] One or more.
[D] Zero or one.
[E] Zero or more.

E

- Week 5: Slides 12, 15
- A negation-free program has exactly one stable model.
- $\{a \leftarrow \text{not } a.\}$ has zero stable models.
- $\{a \leftarrow \text{not } b. \quad b \leftarrow \text{not } a.\}$ as two models.

Question 11

ASP offers several extensions of normal logic programs. Which of the following extensions is **not** supported by ASP?

[A] Integrity constraints, i.e., rules of the form $\leftarrow B_1, \ldots, B_m, \text{not}C_1, \ldots, \text{not}C_n$.
[B] Choice rules, i.e., rules of the form $\{A_1, \ldots, A_k\} \leftarrow B_1, \ldots, B_m, \text{not}C_1, \ldots, \text{not}C_n$.
[C] Rules with a negated head, i.e., rules of the form $\text{not}A \leftarrow B_1, \ldots, B_m, \text{not}C_1, \ldots, \text{not}C_n$.
[D] Disjunctive rules, i.e., rules of the form $A_1; \ldots; A_k \leftarrow B_1, \ldots, B_m, \text{not}C_1, \ldots, \text{not}C_n$.
[E] Fist-order quantification, i.e., rules of the form
$Q_1 x_1 \ldots Q_k x_k \big(A(\vec{x}) \leftarrow B_1(\vec{x}), \ldots, B_m(\vec{x}), \text{not}C_1(\vec{x}), \ldots, \text{not}C_n(\vec{x})\big)$. where $Q_i \in \{\exists, \forall\}$

E

- Slides 19–24

Question 12

Consider the CNF formula $\phi = (\bar{p} \vee q \vee \bar{t}) \wedge (r \vee t) \wedge (\bar{t} \vee p) \wedge (s \vee \bar{r})$ and $I = \{\bar{s}\}$. When I is closed under unit propagation using the watched-literal scheme, which literals are added to I in which order?

[A] That depends on the watched literals in this case.
[B] No literals are added.
[C] The following literals are added: $\bar{r}, t, p, q$
[D] The following literals are added: $\bar{r}, p$
[E] The following literals are added: $\bar{r}$

C

- Week 7: Slides 23–28
- Note that you do not need to use the watched-literal scheme to close I under unit propagation. The watched-literal scheme is just an efficient algorithm for that purpose. When doing unit propagation by hand, you are probably much quicker doing it the naive way as described on Slide 24, for example.
- Unit propagation starts with $\bar{s}$, then obtains $\bar{r}$ from $(s \vee \bar{r})$, then obtains t from $(r \vee t)$, then obtains p from $(\bar{t} \vee p)$, and finally obtains q from $(\bar{p} \vee q \vee \bar{t})$.
- Note that the order is uniquely determined in this example. This is because after adding a literal no more than one new literal is derived by unit propagation. If we replace $(\bar{t} \vee p)$ with $(r \vee p)$, then the order is not uniquely determined anymore because once $\bar{r}$ is added, both p (from $(r \vee p)$) and t (from $(r \vee t)$) follow, so $\bar{r}, t, p, q$ is one possible order, $\bar{r}, p, t, q$ is another.

Question 13

Which of the following sentences is valid for arbitrary ϕ?

[A] $\neg\mathbf{K}\phi \rightarrow \mathbf{K}\neg\mathbf{K}\phi$
[B] $\mathbf{K}\phi \vee \mathbf{K}\neg\phi$
[C] $\neg\mathbf{K}\phi \rightarrow \mathbf{K}\neg\phi$
[D] $\mathbf{K}\phi \rightarrow \neg\mathbf{K}\neg\phi$
[E] $\neg(\mathbf{K}\phi \wedge \mathbf{K}\neg\phi)$

A

- A *valid* formula is satisfied in all interpretations.
- Recall that e, w satisfies $\neg\mathbf{K}\phi$ iff for some $w' \in e$, e, w' satisfies $\neg\phi$ iff for some $w' \in e$, e, w' falsifies ϕ.
- [A] says that if ϕ is not known, then it is known that ϕ is not known. More formally, it says that if for some $w' \in e$, e, w' falsifies ϕ, then for all $w'' \in e$ there is a $w''' \in e$ such that e, w''' falsifies ϕ, which is true.
- [B] says that either ϕ is known or $\neg\phi$ is known, which is not true in general (it would mean that we have complete knowledge).
- [C] says that if ϕ is not known, then $\neg\phi$ is known. More formally, it says that for some $w' \in e$, e, w' falsifies ϕ, then for all $w'' \in e$, e, w'' falsifies ϕ, which is not true because different possible worlds may disagree on ϕ.
- [D] says that if ϕ is known, then $\neg\phi$ is not known. This does hold for interpretations e, w where the set of possible worlds e is non-empty. If e is empty, however, every formula is known, including ϕ as well as $\neg\phi$.

- [E] says that it is impossible to know ϕ and know $\neg\phi$. Using De Morgan's rules, we can rephrase it as $\neg\mathbf{K}\phi \vee \neg\mathbf{K}\neg\phi$, which says that either ϕ is not known or $\neg\phi$ is not known. As in [D], consider an interpretation e, w where $e = \{\}$. Then e, w satisfies $\mathbf{K}\phi$ as well as $\mathbf{K}\neg\phi$.

Question 14

What does the frame problem refer to?

[A] Representing the positive effects of actions.
[B] Representing the negative effects of actions.
[C] Representing what is not changed by actions.
[D] Representing the minor preconditions of an action.
[E] Representing the indirect effects of an action.

C

- Week 8: Slide 27
- The Frame problem: How to represent what *doesn't* change. E.g., how do I represent that when I pick up a cup, all the other items on my desk do not move?
- Note: [D] is about the part of the *Qualification* problem, [E] is the *Ramification* problem.

Question 15

Which statement is true?
[A] A Condorcet winner always exists.
[B] The Borda winner is the Condorcet winner.
[C] The plurality winner is the Condorcet winner.
[D] The Condorcet winner is the Borda winner.
[E] None of the above.

E

- Lecture 9 on social choice.

Question 16

Which statement is false?
[A] Probabilistic Serial mechanism is strategyproof.
[B] When allocating more than one item per agent, sequential allocation is not strategyproof.
[C] Random serial dictatorship is strategyproof.
[D] Random dictatorship is strategyproof.
[E] None.

A

- Lecture 10 on resource allocation.

Question 17

Which of the cooperative game solutions always lies within the least core?

[A] Core.
[B] Nucleolus.
[C] ϵ-core.
[D] Shapley value.
[E] None of the above.

B

Question 18

In the game theory problem of the Prisoner's Dilemma, what is the Nash Equilibria

[A] A mixed Nash Equilibria of each player choosing to defect with probability 1/2.
[B] A pure Nash Equilibria of both players co-operating.
[C] A pure Nash Equilibria of both players defecting.
[D] A pure Nash Equilibria of one player defecting and the other co-operating.
[E] There is no Nash Equilibria.

C

- Week 12: Noncooperative Games, slide 19.

Question 19

Consider the following profile with 10 voters and 3 candidates. E.g., there are 4 voters with preference A $\succ$ B $\succ$ C.

4	3	3
A	B	C
B	C	A
C	A	B

What are the Borda scores for each candidate?

[A] A:10, B:11, C:7.
[B] A:10, B:8, C:7.

[C] A:11, B:9, C:8.
[D] A:11, B:10, C:9.
[E] None of the above.

D

- Week 9: Social Choice.
- Score for A: $4 \times 2 + 3 \times 1 = 11$;
- Score for B: $4 \times 1 + 3 \times 2 = 10$;
- Score for C: $3 \times 1 + 3 \times 2 = 9$;

Question 20

The top trading cycles algorithm does not satisfy which of the properties?

[A] Individual rationality.
[B] Core stability.
[C] Strategyproofness.
[D] Envy-freeness.
[E] Pareto optimality.

D

Part B: Introduction to KRR, Formal Logic and Reasoning

NOTE: Answer the questions in this section in the answer book provided.

Make your answers as clear and easy to understand as possible. Confusing or illegible solutions will lose marks.

Question 21

(8 marks)

Determine whether the following hold:

- $p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$
- $p \rightarrow q \models \neg q \rightarrow \neg p$
- $\models p \leftrightarrow \neg\neg p$
- $p \vee \neg p \vdash$

- Since this uses the single turnstile $\vdash$ we need to use a syntactic method like resolution. Here is a possible proof:
  1. $\neg p \vee q$ Premise (converted to CNF)
  2. $\neg q \vee r$ Premise (converted to CNF)
  3. p Negation of conclusion (converted to CNF)
  4. $\neg r$ Negation of conclusion (converted to CNF)
  5. q 1, 3 Resolution
  6. r 2, 5 Resolution
  7. $\Box$ 5, 6 Resolution

- Since this uses the double turnstile $\models$ we need to use a semantic method like truth tables. Here is a possible proof:

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

  In all rows where $p \rightarrow q$, $\neg q \rightarrow \neg p$ is also true therefore the entailment holds.

-

p	$p \leftrightarrow \neg\neg p$
T	T
F	T

  The last column is always true so this entailment holds.

- This sequent has no meaning although you could argue that the righ-hand side could be replaced by true.

Question 22

(6 marks)

Consider the following two sentences:

[A] All birds except emu's fly

[B] Tweety is a bird that doesn't fly

Write a formula in first-order logic expressing each of the given facts. Call them A and B. Show semantically whether these two formulas are sufficient to determine whether Tweety is an emu or not.

A: $\forall x.(bird(x) \wedge \neg emu(x)) \rightarrow fly(x)$
B: $bird(Tweety) \wedge \neg fly(Tweety)$
Proof:
Consider any interpretation I that satisfies both A and B (i.e., $I \models A$ and $I \models B$).
Let's suppose that Tweety is not an emu (i.e., $I \models \neg emu(Tweety)$).
From B we know that $I \models bird(Tweety)$.
Therefore, we have that $I \models bird(Tweety) \wedge \neg emu(Tweety)$.
Then, from A, we know that $I \models fly(Tweety)$.
However, from B we also know that $I \models \neg fly(Tweety)$.
This is a contradiction therefore our assumption was incorrect and $I \models emu(Tweety)$. That is, Tweety is an emu.

Question 23

(6 marks)

Determine whether the following is a valid inference in first-order logic using resolution:
$\forall x.(P(x) \to Q(x)), \forall x.(\neg R(x) \to \neg Q(x)) \vdash \forall x.(\neg R(x) \to \neg P(x))$

CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \lor Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \lor Q(x)$ (Drop $\forall$)
CNF($\forall x.(\neg R(x) \to \neg Q(x))$)
$\equiv \forall x.(\neg\neg R(x) \lor \neg Q(x))$ (Remove $\to$)
$\equiv \forall x.(R(x) \lor \neg Q(x))$ (Remove $\neg\neg$)
$\equiv R(x) \lor \neg Q(x)$ (Remove $\forall$)

CNF($\neg\forall x.(\neg R(x) \to \neg P(x))$) (Negation of conclusion)
$\equiv \exists x.\neg(\neg R(x) \to \neg P(x))$ ($\neg\forall \equiv \exists\neg$ drive negation inwards)
$\equiv \exists x.\neg(\neg\neg R(x) \lor \neg P(x))$ (Remove $\to$)
$\equiv \exists x.\neg\neg\neg R(x) \land \neg\neg P(x)$ (De Morgan's Law)
$\equiv \exists x.\neg R(x) \land P(x)$ (Remove $\neg\neg$)
$\equiv \neg R(a) \land P(a)$ (Skolemisation)
Proof:
1. $\neg P(x) \lor Q(x)$ Premise
2. $R(x) \lor \neg Q(x)$ Premise
3. $\neg R(a)$ Negation of conclusion
4. $P(a)$ Negation of conclusion
5. $Q(a)$ 1. ($\{x/a\}$), 4 Resolution
6. $R(a)$ 2. ($\{x/a\}$), 5 Resolution
7. $\Box$ 3, 6 Resolution

Therefore, yes, this is a valide inference.

Part C: Non-monotonic reasoning, reasoning about knowledge, reasoning about actions

NOTE: Answer the questions in this section in the answer book provided.

Make your answers as clear and easy to understand as possible. Confusing or illegible solutions will lose marks.

Question 24

(10 marks)

Determine all stable models of the following ASP program P:

$$
\begin{aligned}
a &\leftarrow c, \text{not } b.\\
b &\leftarrow c, \text{not } a.\\
c &\leftarrow \text{not } a.\\
c &\leftarrow \text{not } b.
\end{aligned}
$$

Use the following table for your solution:

S	P^S	Stable?
$\{a, b, c\}$		
$\{a, b\}$		
$\{a, c\}$		
$\{b, c\}$		
$\{a\}$		
$\{b\}$		
$\{c\}$		
$\{\}$		

S	P^S	Stable?
$\{a, b, c\}$		no
$\{a, b\}$		no
$\{a, c\}$	$a \leftarrow c.$ $c.$	yes
$\{b, c\}$	$b \leftarrow c.$ $c.$	yes
$\{a\}$	$a \leftarrow c.$ $c.$	no
$\{b\}$	$b \leftarrow c.$ $c.$	no
$\{c\}$	$a \leftarrow c.$ $b \leftarrow c.$ $c.$ $c.$	no
$\{\}$	$a \leftarrow c.$ $b \leftarrow c.$ $c.$ $c.$	no

Question 25

(10 marks)

Consider the following scenario: a robot can put items into a single storage box and take them out of it. In the Logic of Actions, we can represent this scenario using

- a predicate $\text{InBox}(x)$ that represents whether or not x is in the storage box;

- an action $\mathrm{putIn}(x)$ that puts object x into the storage box;
- an action $\mathrm{takeOut}(x)$ that takes object x out of the storage box.

[A] Write a successor-state axiom for $\mathrm{InBox}(x)$.

[B] Determine $\mathcal{R}([\mathrm{takeOut}(\mathrm{Book})]\neg\mathrm{InBox}(\mathrm{Book}))$.

[A] The successor-state axiom is

$$\Box\forall a\forall x([a]\mathrm{InBox}(x) \leftrightarrow a = \mathrm{putIn}(x) \vee (\mathrm{InBox}(x) \wedge a \neq \mathrm{takeOut}(x)))$$

It can be derived from the effect axioms:

$$\Box\forall a\forall x\big(a = \mathrm{putIn}(x) \to [a]\mathrm{InBox}(x)\big)$$
$$\Box\forall a\forall x\big(a = \mathrm{takeOut}(x) \to [a]\neg\mathrm{InBox}(x)\big)$$

[B] The regression works as follows:

$$\begin{aligned}
&\mathcal{R}\big([\mathrm{takeOut}(\mathrm{Book})]\neg\mathrm{InBox}(\mathrm{Book})\big)\\
&= \neg\mathcal{R}\big([\mathrm{takeOut}(\mathrm{Book})]\mathrm{InBox}(\mathrm{Book})\big)\\
&= \neg\mathcal{R}\big({\gamma_{\mathrm{Book}}}^{a}_{\mathrm{takeOut}(\mathrm{Book})}{}^{x}_{\mathrm{Book}}\big)\\
&= \neg\mathcal{R}\big(\mathrm{takeOut}(\mathrm{Book}) = \mathrm{putIn}(\mathrm{Book}) \vee (\mathrm{InBox}(\mathrm{Book}) \wedge \mathrm{takeOut}(\mathrm{Book}) \neq \mathrm{takeOut}(\mathrm{Book}))\big)\\
&= \neg\big(\mathcal{R}(\mathrm{takeOut}(\mathrm{Book}) = \mathrm{putIn}(\mathrm{Book})) \vee \mathcal{R}(\mathrm{InBox}(\mathrm{Book}) \wedge \mathrm{takeOut}(\mathrm{Book}) \neq \mathrm{takeOut}(\mathrm{Book}))\big)\\
&= \neg\big(\mathcal{R}(\mathrm{takeOut}(\mathrm{Book}) = \mathrm{putIn}(\mathrm{Book})) \vee (\mathcal{R}(\mathrm{InBox}(\mathrm{Book})) \wedge \mathcal{R}(\mathrm{takeOut}(\mathrm{Book}) \neq \mathrm{takeOut}(\mathrm{Book})))\big)\\
&= \neg\big(\mathrm{takeOut}(\mathrm{Book}) = \mathrm{putIn}(\mathrm{Book}) \vee (\mathrm{InBox}(\mathrm{Book}) \wedge \mathrm{takeOut}(\mathrm{Book}) \neq \mathrm{takeOut}(\mathrm{Book}))\big)
\end{aligned}$$

It is not necessary to write down all intermediate steps: full marks would be given here even if the second and third from last lines were missing.

Optionally, the formula can be simplified further using De Morgan's laws:

$$(\mathrm{takeOut}(\mathrm{Book}) \neq \mathrm{putIn}(\mathrm{Book}) \wedge (\neg\mathrm{InBox}(\mathrm{Book}) \vee \mathrm{takeOut}(\mathrm{Book}) = \mathrm{takeOut}(\mathrm{Book})))$$

which further simplifies to

$$(\neg\mathrm{InBox}(\mathrm{Book}) \vee \mathrm{takeOut}(\mathrm{Book}) = \mathrm{takeOut}(\mathrm{Book}))$$

which is equivalent to TRUE.

Part D: Decision Making

NOTE: Answer the questions in this section in the file answer book provided.

Make your answers as clear and easy to understand as possible. Confusing or illegible solutions will lose marks. Provide justifications where needed but irrelevant text detracting from the answer will lose marks.

Question 26

(10 marks)

Consider the following school choice problem with four students 1, 2, 3, 4 and four schools a, b, c, and d with each school having exactly one seat. The preferences of the students are as follows.

$$1 : b \succ a \succ c \succ d$$
$$2 : a \succ b \succ c \succ d$$
$$3 : a \succ b \succ c \succ d$$
$$4 : d \succ b \succ c \succ a$$

The preferences of the schools are as follows.

$$a : 1 \succ 3 \succ 2 \succ 4$$
$$b : 2 \succ 1 \succ 3 \succ 4$$
$$c : 2 \succ 1 \succ 3 \succ 4$$
$$d : 4 \succ 1 \succ 3 \succ 2$$

Find the outcome matching of the student proposing deferred acceptance algorithm and explain how you found the matching. Prove or disprove that the resultant matching is Pareto optimal for the students.

- In round 1, 1 proposes to b; 2 and 3 proposes to a; and 4 proposes to d. The tentative match is $\{(1, b), (3, a), (4, d)\}$ as student 2 is rejected from a.
- Student 2 then proposes to b. The new match is $\{(2, b), (3, a), (4, d)\}$ as student 1 is rejected by b.
- Student 1 then proposes to a. The tentative match is $\{(1, a), (2, b), (4, d)\}$ as student 3 is rejected by a.
- Student 3 proposes to b. The tentative match is $\{(1, a), (2, b), (4, d)\}$ as student 3 is rejected by b.
- Student 3 proposes to b. The final match is $\{(1, a), (2, b), (3, c), (4, d)\}$.

The matching is not Pareto optimal for the students as 1 and 2 can exchange their positions to get a more preferred school.

Question 27

(10 marks)

Compute all the Nash equilibria of the following two player game and explain how you computed them.

	D	E
A	2,3	8,5
B	6,6	4,2

We need to find all the NE (Nash equilibria) of the game. As is convention, the row player is player 1 and the column player is player 2. (A,E) and (B,D) are in PNE (pure NE).

Any pure action profile in which one player plays one action and the other player mixes over both actions is not in PNE. Let us consider a possible mixed Nash equilibrium in which both agents mix over over all their actions.

Suppose Player 2 plays D with probability p and E with probability $1-p$. When player 1 is indifferent between her actions

$$2p + 8(1-p) = 6p + 4(1-p)$$
$$p = 1/2$$

Suppose Player 1 plays A with probability q and B with probability 1-q. Player 2 is indifferent between her actions

$$3q + 6(1-q) = 5q + 2(1-q)$$
$$q = 2/3$$

Therefore the strategy profile is in Nash equilibrium: Player 1 plays A with probability 2/3 and B with probability 1/3 and Player 2 plays D with probability 1/2 and E with probability 1/2.