

# Exercise sheet 1b

## COMP6741: Parameterized and Exact Computation

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19T3

**Exercise 1.** An *independent set* of a graph  $G = (V, E)$  is a subset of vertices  $S \subseteq V$  such that no two vertices of  $S$  are adjacent in  $G$ . Consider the INDEPENDENT SET and the MAXIMUM INDEPENDENT SET problems.

INDEPENDENT SET

Input: Graph  $G$ , integer  $k$

Question: Does  $G$  have an independent set of size at least  $k$ ?

MAXIMUM INDEPENDENT SET

Input: Graph  $G$

Output: A largest independent set of  $G$

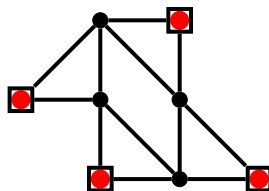


Figure 1: An independent set of size 4 in a graph

1. Given a  $O(1.2^n)$ -time algorithm for MAXIMUM INDEPENDENT SET, design an algorithm for INDEPENDENT SET with running time  $O(1.2^n \cdot \text{poly}(n))$ .
2. Given a  $O(1.2^n)$ -time algorithm for INDEPENDENT SET, design an algorithm for MAXIMUM INDEPENDENT SET with running time  $O(1.2^n \cdot \text{poly}(n))$ .

**Exercise 2.** Discussion topic.

Are Nondeterministic Turing Machines realistic computation models?

- Is this a good representation of how our computing devices work?
- What is different?

What about Deterministic Turing Machines?

**Exercise 3.** Design a Deterministic Turing Machine  $(Q, \Gamma, \Sigma = \{0, 1\}, q_0, A, \delta)$  that accepts palindromes. A *palindrome* is a word that is equal to its reverse; e.g., 011010110.

**Exercise 4.** A *vertex cover* in a graph  $G = (V, E)$  is a subset of vertices  $S \subseteq V$  such that every edge of  $G$  has an endpoint in  $S$ .

VERTEX COVER

Input: Graph  $G$ , integer  $k$

Question: Does  $G$  have a vertex cover of size  $k$ ?

- Prove that VERTEX COVER is NP-complete.