COMP4418: Knowledge Representation and Reasoning—Solutions to Exercise 1 Propositional Logic

1. (i) $(\neg Ja \wedge \neg Jo) \rightarrow T$
   Where:
   Ja: *Jane is in town*
   Jo: *John is in town*
   T: *we will play tennis*

   (ii) $R \vee \neg R$
   Where:
   R: *it will rain today*

   (iii) $\neg S \rightarrow \neg P$
   Where:
   S: *you study*
   P: *you will pass this course*

2. (i) $P \rightarrow Q$
   $\neg P \vee Q$ (remove $\rightarrow$)

   (ii) $(P \rightarrow \neg Q) \rightarrow R$
   $\neg(\neg P \vee \neg Q) \vee R$ (remove $\rightarrow$)
   $(\neg\neg P \wedge \neg\neg Q) \vee R$ (De Morgan)
   $(P \wedge Q) \vee R$ (Double Negation)
   $(P \vee R) \wedge (Q \vee R)$ (Distribute $\vee$ over $\wedge$)

   (iii) $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$
   $\neg\neg(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (remove $\rightarrow$)
   $(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (Double Negation)
   $(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R \vee \neg Q)$ (Distribute $\vee$ over $\wedge$)
   This can be further simplified to: $((P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R)$
   And in fact this cab be simplified to $\neg Q \vee \neg R$ since $(\neg Q \vee \neg R) \vdash (P \vee \neg R \vee \neg Q)$

3. (i)

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

   In all rows where both $P \rightarrow Q$ and $\neg Q$ are true, $\neg P$ is also true. Therefore, inference is valid.

   (ii)

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow \neg P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

   In all rows where both $P \rightarrow Q$ is true, $\neg Q \rightarrow \neg P$ is also true. Therefore, inference is valid.

(iii)

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

In all rows where both $P \to Q$ and $Q \to R$ are true, $P \to R$ is also true. Therefore, inference is valid.

4. (i) CNF($P \to Q$)
$\equiv \neg P \vee Q$

CNF($\neg Q$)
$\equiv \neg Q$

CNF($\neg\neg P$)
$\equiv P$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Hypothesis)
2.	$\neg Q$	(Hypothesis)
3.	P	(Negation of Conclusion)
4.	Q	1, 3 Resloution
5.	$\square$	2, 4 Resloution

(ii) CNF($P \to Q$)
$\equiv \neg P \vee Q$

CNF($\neg(\neg Q \to \neg P)$)
$\equiv \neg(\neg\neg Q \vee \neg P)$ (Remove $\to$)
$\equiv \neg(Q \vee \neg P)$ (Double Negation)
$\equiv \neg Q \wedge \neg\neg P$ (De Morgan)
$\equiv \neg Q \wedge P$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Hypothesis)
2.	$\neg Q$	(Negation of Conclusion)
3.	P	(Negation of Conclusion)
4.	$\neg P$	1, 2 Resolution
5.	$\square$	3, 4 Resolution

(iii) $P \to Q, Q \to R \vdash P \to R$
CNF($P \to Q$)
$\equiv \neg P \vee Q$

CNF($Q \to R$)
$\equiv \neg Q \vee R$

CNF($\neg(P \to R)$)
$\equiv \neg(\neg P \vee R)$ (Remove $\to$)
$\equiv \neg\neg P \wedge \neg R$ (De Morgan)
$\equiv P \wedge \neg R$ (Double Negation)

Proof:

1.	$\neg P \vee Q$	(Hypothesis)
2.	$\neg Q \vee R$	(Hypothesis)
3.	P	(Negation of Conclusion)
4.	$\neg R$	(Negation of Conclusion)
5.	Q	1, 3 Resoltion
6.	R	2, 5 Resolution
7.	$\square$	4, 6 Resolution

5. (i)

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \to Q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P and Q. Therefore $((P \vee Q) \wedge \neg P) \to Q$ is a tautology.

(ii) $((P \to Q) \wedge \neg(P \to R)) \to (P \to Q)$

(iii)

P	Q	R	$P \to Q$	$\neg(P \to R)$	$(P \to Q) \wedge \neg(P \to R)$	$((P \to Q) \wedge \neg(P \to R)) \to (P \to Q)$
T	T	T	T	F	F	T
T	T	F	T	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T
F	F	F	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P, Q and R. Therefore $((P \to Q) \wedge \neg(P \to R)) \to (P \to Q)$ is a tautology.

(iv)

P	$\neg P$	$\neg P \wedge P$	$\neg(\neg P \wedge P)$	$\neg(\neg P \wedge P) \wedge P$
T	F	F	T	T
F	T	F	T	F

Last column is not always true. Therefore $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(v) $(P \vee Q) \to \neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$(P \vee Q) \to \neg(\neg P \wedge \neg Q)$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

6. (i) CNF($\neg(((P \vee Q) \wedge \neg P) \rightarrow Q)) \equiv \neg(\neg((P \vee Q) \wedge \neg P) \vee Q)$ (Remove $\rightarrow$)
$\equiv \neg\neg((P \vee Q) \wedge \neg P) \wedge \neg Q)$ (DeMorgan)
$\equiv (P \vee Q) \wedge \neg P) \wedge \neg Q$ (Double Negation)

Proof:

1.	$P \vee Q$	(Negated Conclusion)
2.	$\neg P$	(Negated Conclusion)
3.	$\neg Q$	(Negated Conclusion)
4.	Q	1, 2 Resolution
5.	$\Box$	3, 4 Resolution

Therefore $\neg(((P \vee Q) \wedge \neg P) \rightarrow Q)$ is a tautology.

(ii) CNF($\neg(((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)))$
$\equiv \neg(\neg((\neg P \vee Q) \wedge \neg(\neg P \vee R)) \vee (\neg P \vee Q))$ (Remove $\rightarrow$)
$\equiv \neg\neg((\neg P \vee Q) \wedge \neg(\neg P \vee R)) \wedge \neg(\neg P \vee Q)$ (De Morgan)
$\equiv (\neg P \vee Q) \wedge (\neg\neg P \wedge \neg R) \wedge (\neg\neg P \wedge \neg Q)$ (Double Negation and De Morgan)
$\equiv (\neg P \vee Q) \wedge (P \wedge \neg R) \wedge (P \wedge \neg Q)(DoubleNegation)$

Proof:

1.	$\neg P \vee Q$	(Negated Conclusion)
2.	P	(Negated Conclusion)
3.	$\neg R$	(Negated Conclusion)
4.	$\neg Q$	(Negated Conclusion)
5.	Q	1, 2 Resolution
6.	$\Box$	4, 5 Resolution

Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

(iii) CNF($\neg(\neg(\neg P \wedge P) \wedge P))$
$\equiv \neg\neg(\neg P \wedge P) \vee \neg P$ (De Morgan)
$\equiv (\neg P \wedge P) \vee \neg P$ (Double Negation)
$\equiv (\neg P \vee \neg P) \vee (P \vee \neg P)$ (Distribute $\wedge$ over $\vee$)
$\equiv \neg P$ (Can simplify to this by removing repetition and tautologies)

Proof:

1.	$\neg P$	(Negated Conclusion)

Cannot obtain empty clause using resolution so $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(iv) CNF($\neg((P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q))) \equiv \neg(\neg(P \vee Q) \vee \neg(\neg P \wedge \neg Q))$ (Remove $\rightarrow$)
$\equiv \neg\neg(P \vee Q) \vee \neg\neg(\neg P \wedge \neg Q))$ (De Morgan)
$\equiv (P \vee Q) \vee (\neg P \wedge \neg Q))$ (Double Negation)

Proof:

1.	$(P \vee Q)$	(Negated Conclusion)
2.	$\neg Q$	(Negated Conclusion)
3.	$\neg P$	(Negated Conclusion)
4.	Q	1, 2 Resolution
5.	$\Box$	3, 4, Resolution

Therefore $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$ is a tautology.