# COMP9444
# Neural Networks and Deep Learning

# 9a. Autoencoders

Textbook, Chapter 14





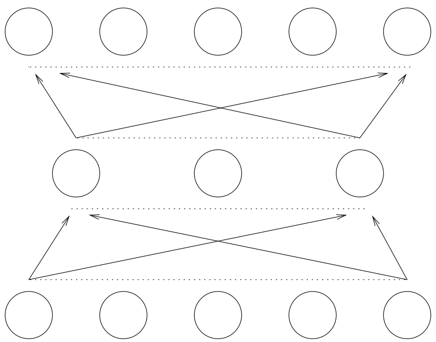
## Outline

- Autoencoder Networks (14.1)
- Regularized Autoencoders (14.2)
- Stochastic Encoders and Decoders (14.4)
- Generative Models
- Variational Autoencoders (20.10.3)





## Recall: Encoder Networks



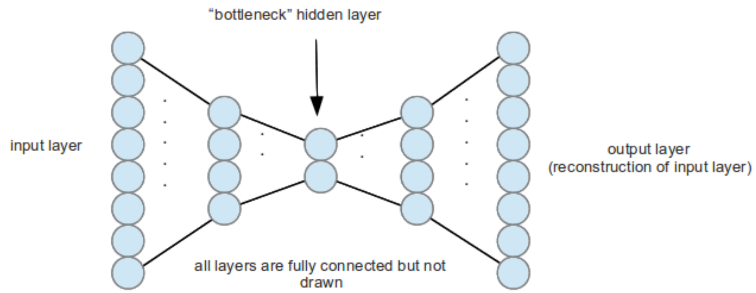
| Inputs | Outputs |
|---|---|
| 10000 | 10000 |
| 01000 | 01000 |
| 00100 | 00100 |
| 00010 | 00010 |
| 00001 | 00001 |

- identity mapping through a bottleneck
- also called N–M–N task
- used to investigate hidden unit representations



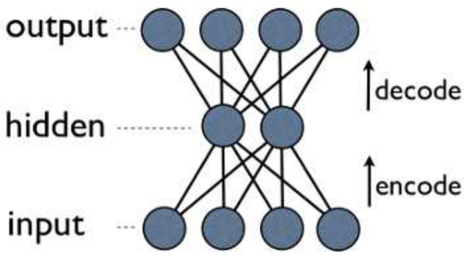


## Autoencoder Networks




- output is trained to reproduce the input as closely as possible
- activations normally pass through a bottleneck, so the network is forced to compress the data in some way
- like the RBM, Autoencoders can be used to automatically extract abstract features from the input

# Autoencoder Networks




If the encoder computes $z = f(x)$ and the decoder computes $g(f(x))$ then we aim to minimize some distance function between $x$ and $g(f(x))$

$$E = L\big(x, g(f(x))\big)$$






# Autoencoder as Pretraining

- after an autoencoder is trained, the decoder part can be removed and replaced with, for example, a classification layer
- this new network can then be trained by backpropagaiton
- the features learned by the autoencoder then serve as initial weights for the supervised learning task






# Greedy Layerwise Pretraining

- Autoencoders can be used as an alternative to Restricted Bolzmann Machines, for greedy layerwise pretraining.
- An autoencoder with one hidden layer is trained to reconstruct the inputs. The first layer (encoder) of this network becomes the first layer of the deep network.
- Each subsequent layer is then trained to reconstruct the previous layer.
- A final classification layer is then added to the resulting deep network, and the whole thing is trained by backpropagation.






# Avoiding Trivial Identity

- If there are more hidden nodes than inputs (which often happens in image processing) there is a risk the network may learn a trivial identity mapping from input to output.
- We generally try to avoid this by introducing some form of regularization.

# Regularized Autoencoders (14.2)

- autoencoders with dropout at hidden layer(s)
- sparse autoencoders
- contractive autoencoders
- denoising autoencoders





# Sparse Autoencoder (14.2.1)

- One way to regularize an autoencoder is to include a penalty term in the loss function, based on the hidden unit activations.
- This is analagous to the weight decay term we previously used for supervised learning.
- One popular choice is to penalize the sum of the absolute values of the activations in the hidden layer

$$E = L(x, g(f(x)) + \lambda \sum_i |h_i|$$

- This is sometimes known as $L_1$-regularization (because it involves the absolute value rather than the square); it can encourage some of the hidden units to go to zero, thus producing a sparse representation.





# Contractive Autoencoder (14.2.3)

- Another popular penalty term is the $L_2$-norm of the derivatives of the hidden units with respect to the inputs

$$E = L(x, g(f(x)) + \lambda \sum_i ||\nabla_x h_i||^2$$

- This forces the model to learn hidden features that do not change much when the training inputs $x$ are slightly altered.





# Denoising Autoencoder (14.2.2)

Another regularization method, similar to contractive autoencoder, is to add noise to the inputs, but train the network to recover the original input

repeat:
  sample a training item $x^{(i)}$
  generate a corrupted version $\tilde{x}$ of $x^{(i)}$
  train to reduce $E = L\big(x^{(i)}, g(f(\tilde{x}))\big)$
end

## Loss Functions and Probability

- We saw previously how the loss (cost) function at the output of a feedforward neural network (with parameters $\theta$) can be seen as defining a probability distribution $p_\theta(x)$ over the outputs. We then train to maximize the log of the probability of the target values.
  - squared error assumes an underlying Gaussian distribution, whose mean is the output of the network
  - cross entropy assumes a Bernoulli distribution, with probability equal to the output of the network
  - softmax assumes a Boltzmann distribution





## Stochastic Encoders and Decoders (14.4)

- For autoencoders, the decoder can be seen as defining a conditional probability distribution $p_\theta(x|z)$ of output $x$ for a certain value $z$ of the hidden or "latent" variables.
- In some cases, the encoder can also be seen as defining a conditional probability distribution $q_\phi(z|x)$ of latent variables $z$ based on an input $x$.
- We have seen an example of this with the Restricted Boltzmann Machine, where $q_\phi(z|x)$ and $p_\theta(x|z)$ are Bernoulli distributions.





## Generative Models

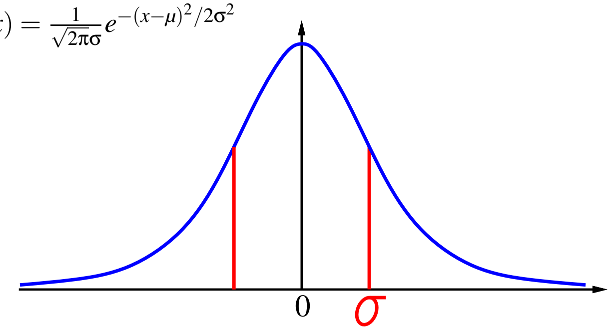
- Sometimes, as well as reproducing the training items $\{x^{(i)}\}$, we also want to be able to use the decoder to generate new items which are of a similar "style" to the training items.
- In other words, we want to be able to choose latent variables $z$ from a standard Normal distribution $p(z)$, feed these values of $z$ to the decoder, and have it produce a new item $x$ which is somehow similar to the training items.
- Generative models can be:
  - explicit (Variational Autoencoders)
  - implicit (Generative Adversarial Networks)





## Gaussian Distribution (3.9.3)

$P_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$




$\mu$ = mean

$\sigma$ = standard deviation

Multivariate Gaussian: $P_{\mu,\sigma}(x) = \prod_i P_{\mu_i,\sigma_i}(x_i)$

# Entropy and KL-Divergence

- The entropy of a distribution $q()$ is $H(q) = \int_\theta q(\theta)(-\log q(\theta))d\theta$
- In Information Theory, H($q$) is the amount of information (bits) required to transmit a random sample from distribution $q()$
- For a Gaussian distribution, $H(q) = \sum_i \log \sigma_i$
- KL-Divergence $D_{\mathrm{KL}}(q \,\|\, p) = \int_\theta q(\theta)(\log q(\theta) - \log p(\theta))d\theta$
- $D_{\mathrm{KL}}(q \,\|\, p)$ is the number of extra bits we need to trasmit if we designed a code for $p()$ but the samples are drawn from $q()$ instead.
- If $p(z)$ is Standard Normal distribution, minimizing $D_{\mathrm{KL}}\big(q_\phi(z)\|p(z)\big)$ encourages $q_\phi()$ to center on zero and spread out to approximate $p()$.





# Variational Autoencoder (20.10.3)

Instead of producing a single $z$ for each $x^{(i)}$, the encoder (with parameters $\phi$) can be made to produce a mean $\mu_{z|x^{(i)}}$ and standard deviation $\sigma_{z|x^{(i)}}$

This defines a conditional (Gaussian) probability distribution $q_\phi(z|x^{(i)})$

We then train the system to maximize

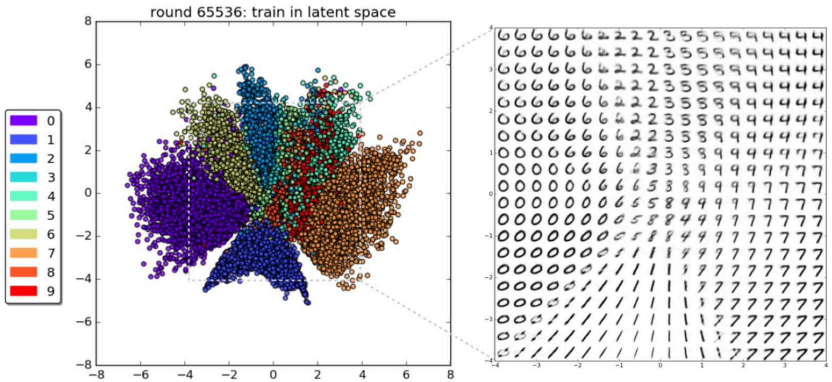
$$\mathbf{E}_{z\sim q_\phi(z|x^{(i)})}[\log p_\theta(x^{(i)}|z)] \;-\; D_{\mathrm{KL}}\big(q_\phi(z|x^{(i)})\|p(z)\big)$$

- the first term enforces that any sample $z$ drawn from the conditional distribution $q_\phi(z|x^{(i)})$ should, when fed to the decoder, produce somthing approximating $x^{(i)}$
- the second term encourages $q_\phi(z|x^{(i)})$ to approximate $p(z)$
- in practice, the distributions $q_\phi(z|x^{(i)})$ for various $x^{(i)}$ will occupy complementary regions within the overall distribution $p(z)$





# Variational Autoencoder Digits








# Variational Autoencoder Digits


1st Epoch


9th Epoch


Original

# Variational Autoencoder Faces





# Variational Autoencoder

- Variational Autoencoder produces reasonable results
- tends to produce blurry images
- often end up using only a small number of the dimensions available to $z$





# References

http://kvfrans.com/variational-autoencoders-explained/

http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture13.pdf

https://arxiv.org/pdf/1606.05908.pdf