COMP4418: Knowledge Representation—Solutions to Exercise Set 2
First-Order Logic

1. (i) All birds fly
   (If an object x is a bird, then it flies.)

   (ii) Everyone has a mother

   (iii) There is someone who is everyone's mother

2. (i) $\forall x.(cat(x) \rightarrow mammal(x))$

   (ii) $\neg\exists x.(cat(x) \wedge reptile(x))$
   or, equivalently, $\forall x.(cat(x) \rightarrow \neg reptile(x))$

   (iii) $\forall x.\exists y.(computer\_scientist(x) \rightarrow likes(x, y))$

3. (i) CNF($\forall x.(bird(x) \rightarrow flies(x))$)
   $\equiv \forall x.(\neg bird(x) \vee flies(x))$ (Remove $\rightarrow$)
   $\equiv \neg bird(x) \vee flies(x)$ (Drop $\forall$))

   (ii) CNF($\exists x.\forall y.\forall z.(person(x) \wedge ((likes(x, y) \wedge yneqz) \rightarrow \neg likes(x, z)))$)
   $\equiv \exists x.\forall y.\forall z.(person(x) \wedge (\neg(likes(x, y) \wedge y \neq z) \vee \neg likes(x, z)))$ (Remove $\rightarrow$)
   $\equiv \exists x.\forall y.\forall z.(person(x) \wedge (\neg likes(x, y) \vee y = z \vee \neg likes(x, z)))$ (De Morgan)
   $\equiv \forall y.\forall z.(person(x) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z)))$ (Skolemisation—c is a constant(
   $\equiv person(c) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z))$ (Drop $\forall$)

4. (i) CNF($\forall x.(P(x) \rightarrow Q(x))$)
   $\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
   $\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

   CNF($\neg\forall x.(\neg Q(y) \rightarrow \neg P(y))$)
   $\equiv \neg\forall x.(\neg\neg Q(y) \vee \neg P(y))$ (Remove $\rightarrow$)
   $\equiv \exists x.\neg(\neg\neg Q(y) \vee \neg P(y))$ (De Morgan)
   $\equiv \exists x.\neg(Q(y) \vee \neg P(y))$ (Double Negation)
   $\equiv \exists x.(\neg Q(y) \wedge \neg\neg P(y))$ (De Morgan)
   $\equiv \exists x.(\neg Q(y) \wedge P(y))$ (Double Negation)
   $\equiv \neg Q(c) \wedge P(c)$ (Skolemisation)

   Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\Box$	3, 5 Resloution

(ii) (Works exactly as in (i).)

CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\neg\forall x.(\neg Q(x) \to \neg P(x))$)
$\equiv \neg\forall x.(\neg\neg Q(x) \vee \neg P(x))$ (Remove $\to$)
$\equiv \neg\forall x.(Q(x) \vee \neg P(x))$ (Double Negation)
$\equiv \exists x.\neg(Q(x) \vee \neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \wedge \neg\neg P(x))$ (De Morgan)
$\equiv \exists x.(\neg Q(x) \wedge P(x))$ (Double Negation)
$\equiv \neg Q(c) \wedge \neg P(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(c)$	(Negated Conclusion)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
5.	$\neg P(c)$	2, 4 Resolution
6.	$\square$	3, 5 Resloution

(iii) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($P(a)$)
$\equiv P(a)$

CNF($\neg Q(a)$)
$\equiv \neg Q(a)$

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(a)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1. $\{x/a\}$)
5.	$\neg Q(a)$	2, 4 Resolution
6.	$\square$	3, 5 Resolution

(iv) CNF($\forall x.(P(x) \to Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\to$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\exists x.P(x)$)
$\equiv P(a)$ (Skolemisation)

CNF($\neg\exists x.Q(x)$)
$\equiv \forall x.\neg Q(x)$ (De Morgan)

$\equiv \neg Q(x)$ (Drop $\forall$)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$P(a)$	(Hypothesis)
3.	$\neg Q(y)$	(Negated Conclusion)
4.	$\neg P(a) \vee Q(a)$	(1. $\{x/a\}$)
5.	$Q(a)$	2, 4 Resolution
6.	$\neg Q(a)$ (3. $\{y/a\}$)	
7.	$\square$	5, 6 Resolution

(v) CNF($\forall x.(P(x) \rightarrow Q(x))$)
$\equiv \forall x.(\neg P(x) \vee Q(x))$ (Remove $\rightarrow$)
$\equiv \neg P(x) \vee Q(x)$ (Drop $\forall$)

CNF($\forall x.(Q(x) \rightarrow R(x))$)
$\equiv \forall x.(\neg Q(x) \vee R(x))$ (Remove $\rightarrow$)
$\equiv \neg Q(x) \vee R(x)$ (Drop $\forall$)

CNF($\neg\forall x.(P(x) \rightarrow R(x))$)
$\equiv \neg\forall x.(\neg P(x) \vee R(x))$ (Remove $\rightarrow$)
$\equiv \exists x.(\neg(\neg P(x) \vee R(x))$ (De Morgan)
$\equiv \exists x.(\neg\neg P(x) \wedge \neg R(x))$ (De Morgan)
$\equiv \exists x.(P(x) \wedge \neg R(x))$ (Double Negation)
$\equiv P(c) \wedge \neg R(c)$ (Skolemisation)

Proof:

1.	$\neg P(x) \vee Q(x)$	(Hypothesis)
2.	$\neg Q(y) \vee R(y)$	(Hypothesis)
3.	$P(c)$	(Negated Conclusion)
4.	$\neg R(c)$	(Negated Conclusion)
5.	$\neg P(c) \vee Q(c)$	(1. $\{x/c\}$)
6.	$\neg Q(c) \vee R(c)$	(2. $\{y/c\}$)
7.	$\neg P(c) \vee R(c)$	5, 6 Resolution
8.	$R(c)$	3, 7 Resolution
9.	$\square$	4, 8 Resolution

5. (i) (A) $\exists x.\forall y.(cs(x) \wedge os(y) \wedge likes(x, y))$
(B) $os(Linux)$
(C) $\exists z.(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$

(ii) (A) CNF($\exists x.\forall y.(cs(x) \wedge os(y) \wedge likes(x, y))$)
$\equiv \forall y.(cs(a) \wedge os(y) \wedge likes(a, y))$ (Skolemisation)
$\equiv cs(a) \wedge os(y) \wedge likes(a, y)$ (Drop $\forall$)
(B) CNF($os(Linux)$)
$\equiv os(Linux)$

(C) CNF($\neg\exists z.(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$)
$\equiv \forall z.\neg(cs(z) \wedge os(Linux) \wedge likes(z, Linux))$ (De Morgan Laws)
$\equiv \forall z.(\neg cs(z) \vee \neg os(Linux) \vee \neg likes(z, Linux))$ (De Morgan Laws)
$\equiv \neg cs(z) \vee \neg os(Linux) \vee \neg likes(z, Linux)$ (Drop $\forall$)

(iii)

	1.	$cs(a)$	(Hypothesis A)
	2.	$os(w)$	(Hypothesis A)
	3.	$likes(a, x)$	(Hypothesis A)
	4.	$os(Linux)$	(Hypothesis B)
	5.	$\neg cs(z) \vee \neg os(Linux) \vee \neg likes(z, Linux)$	(Negated Conclusion)
(iii)	6.	$\neg cs(a) \vee \neg os(Linux) \vee \neg likes(a, Linux)$	(5. $\{z/a\}$)
	7.	$\neg os(Linux) \vee \neg likes(a, Linux)$	(1, 6 Resolution)
	8.	$likes(a, Linux)$	(3. $\{x/Linux\}$)
	9.	$\neg os(Linux)$	(7, 8 Resolution)
	10.	$os(Linux)$	(3. {w/Linux})
	11.	$\square$	(9, 10 Resolution)

(iv) Yes. A, B, $\neg C$ in (i) are Horn clauses so there must be an SLD resolution of the empty clause if there is a resolution of the empty clause. In fact, the resolution in (ii) is an SLD resolution of the empty clause.

(v) $A, B \vdash C$