

## 1 Answer Set Programming

### 1.1 Modelling

Let  $S = \{s_1, \dots, s_n\}$  be a set of sets. A *set cover* of  $S$  is a set  $C \subseteq S$  such that  $\bigcup_{s \in S} s = \bigcup_{s \in C} s$ . A *k-set cover* is a set cover of size  $k$ , that is,  $|C| = k$ .

For instance, for an input  $S = \{\{1, 2\}, \{2, 3\}, \{4, 5\}, \{1, 2, 3\}\}$ , there is a 2-set cover  $C = \{\{1, 2, 3\}, \{4, 5\}\}$  since  $\bigcup_{s \in S} s = \{1, 2\} \cup \{2, 3\} \cup \{4, 5\} \cup \{1, 2, 3\} = \{1, 2, 3\} \cup \{4, 5\} = \bigcup_{s \in C} s$ .

Write an ASP program that decides the  $k$ -SET-COVER problem:

Input: a set of sets and a natural number  $k \geq 0$ .

Problem: decide if there is a  $k$ -set cover.

Assume the input parameter  $S = \{s_1, \dots, s_n\}$  is encoded by a binary predicate  $\mathbf{s}$  in the way that  $x \in s_i$  iff  $\mathbf{s}(i, x)$ . The input parameter  $k$  is given as constant symbol  $\mathbf{k}$ . Use a unary predicate  $\mathbf{c}$  to represent the output  $C$  in the way that  $s_i \in C$  iff  $\mathbf{c}(i)$ .

## 1.2 Semantics

Consider the following program  $P$ .

$a.$   
 $c \leftarrow \text{not } b, \text{not } d.$   
 $d \leftarrow a, \text{not } c.$

Determine the stable models of  $S$ .

$S$	Reduct $P^S$	Stable model?
$\{a, b, c, d\}$	$a.$	<b>X</b>
$\{a, b, c\}$		
$\{a, b, d\}$		
$\{a, c, d\}$		
$\{b, c, d\}$		
$\{a, b\}$		
$\{a, c\}$		
$\{a, d\}$		
$\{b, c\}$		
$\{b, d\}$		
$\{c, d\}$		
$\{a\}$		
$\{b\}$		
$\{c\}$		
$\{d\}$		
$\{\}$		