Abstract Interpretation for Code Analysis and Verification

(Week 9)

Yulei Sui

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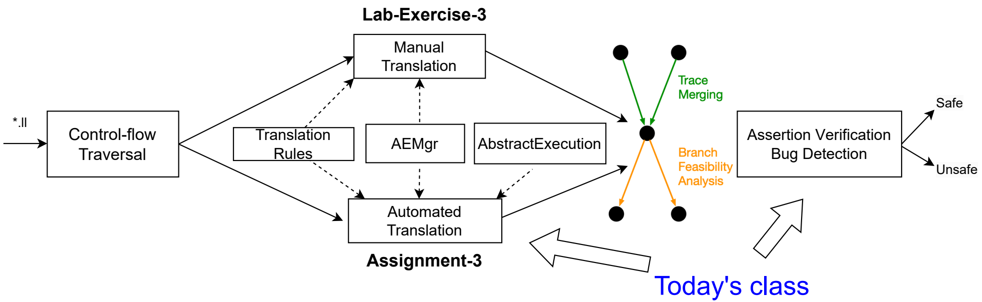
Today's class



- Weak topological order (WTO) and handling loops
- Abstract interpretation over a combined abstract domain

Topological Order

Analysis Order of Nodes on Control-Flow Graph

- ? How to analyze a **loop-free program**?

Topological Order

Analysis Order of Nodes on Control-Flow Graph

? How to analyze a **loop-free program**?

✔ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

Topological Order

Analysis Order of Nodes on Control-Flow Graph

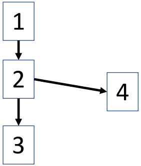
? How to analyze a **loop-free program**?

✓ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

A **topological order** of a graph $G(V, E)$ is a linear ordering of its nodes such that for every directed edge $a \to b$, node a always precedes node b in the ordering.

- Must be a **directed acyclic graph** (DAG), at least one topological ordering.
- The ordering respects the **direction of edges.**

Example of topological order:



acyclic graph G

1 2 3 4 ✓

1 2 4 3 ✓

1 3 2 4 ✗

Valid/invalid topological order

How About Analyzing Loops?

- **Topological Order** can only be used for directed acyclic graphs (DAGs).
- **Weak Topological Order (WTO)** is a relaxation of the more stringent topological order for graphs with loops.
  - **Cycles Permitted**: allows for cycles within the graph.
  - **Hierarchical Decomposition**: A graph is decomposed into a hierarchical structure where each node or a strongly connected component (SCC) can contain subnodes.
  - **Weak Topological Order or Partial Order**: In a WTO, nodes and SCCs are arranged in a partial order (not enumerating possible infinite loop paths). This order respects the dependencies in a way that allows for iterative analysis.
  - We will practice loop handling using WTO in `Assignment-3`.

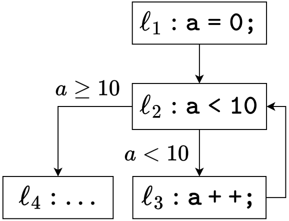
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

? How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph

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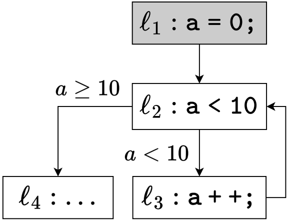
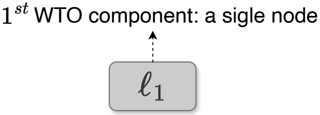


Control Flow Graph



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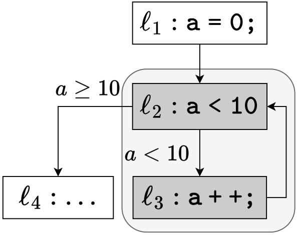

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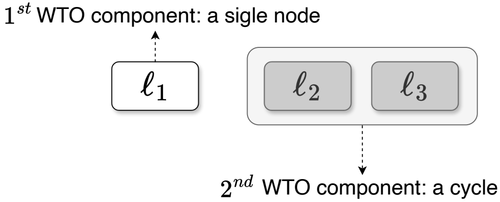

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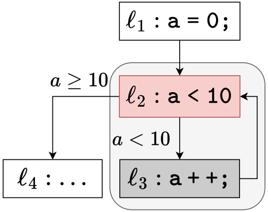

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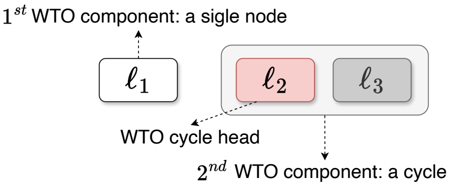

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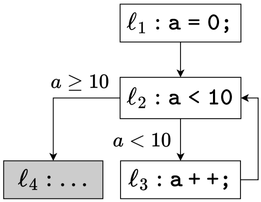

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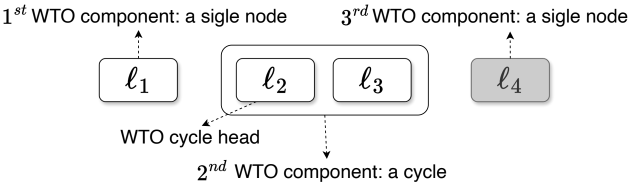

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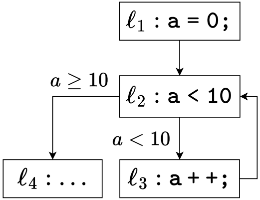

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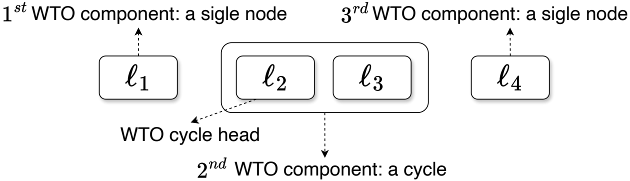

Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

Weak Topological Order

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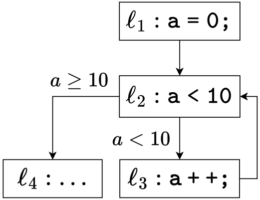

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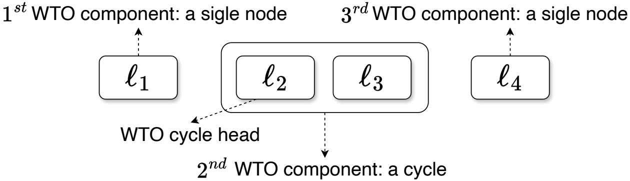

Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

Perform widening when analyzing the cycle heads (i.e., ℓ_2)

WTO, Widening and Narrowing

Why Weak Topological Order (WTO)?

- Handling cyclic dependencies
- Efficient fixed-point computation

Why Widening?

- Over-approximation
- Prevent non-termination

Why Narrowing?

- Refine precision after widening converges
- The specific conditions or constraints used for narrowing:
  - Loop exit conditions (this course)
  - Type constraints (8-bit integer ranging from [-128, 127])
  - Bounds from arithmetic operations If $\texttt{x = y + z}$, and $\texttt{y} \in [1, 5]$ and $\texttt{z} \in [2, 3]$, then $\texttt{x} \in [3, 8]$. If widening gives [1, 10], narrowing can refine this to [3, 8].
  - User-specification (assertions and guard conditions)

Revisit the Notations and Data Structure

- An **abstract trace** $\sigma \in \mathbb{L} \times \mathcal{V} \rightarrow \mathbb{A}$ represents a list of abstract states before ($\overline{\ell}$) and after ($\underline{\ell}$) each program statement ℓ (`preAbsTrace` and `postAbsTrace`).
- An **abstract state** (`AEState` in Lab-3 and Assignment-3) is defined as a map $AS : \mathcal{V} \rightarrow \mathbb{A}$ associating program variables $\mathcal{V}$ with an abstract value in $\mathbb{A}$, approximating the runtime states of program variables.

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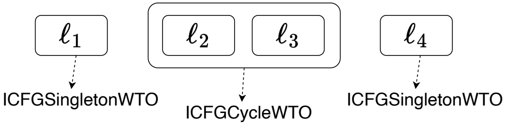
- An **abstract value** can be either an interval or a memory address.

	Domain	Notation	SSE Data Structure
Abstract **trace**	$\mathbb{L} \times \mathcal{V} \to \mathbb{A}$	σ	`preAbsTrace`: **trace** before `ICFGNodes` `postAbsTrace`: **trace** after `ICFGNodes`
Abstract **state** at $L \in \mathbb{L}$	$\mathcal{V} \to \mathbb{A}$	$\sigma_{\overline{\ell}}$ $\sigma_{\underline{\ell}}$	`preAbsTrace[node]`: **state** before `node` ℓ `postAbsTrace[node]`: **state** after `node` ℓ
Abstract **value** of varId at $L \in \mathbb{L}$	$\mathbb{A}$	$\sigma_{\underline{\ell}}$(varId)	`as` $=$ `postAbsTrace[node]` `as[VarID]`: **value** of varId after `node` ℓ

Widening and Narrowing



Analyze ICFGCycleWTO

`ICFGWTOComp` has two subclasses:

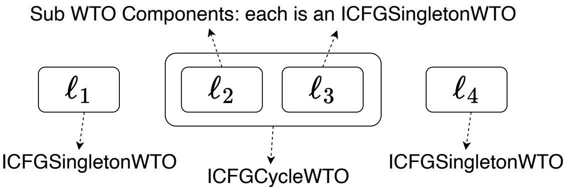
`ICFGSingletonWTO` and `ICFGCycleWTO`.

Use `dyn_cast` to identify the subclass of an instance.

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing



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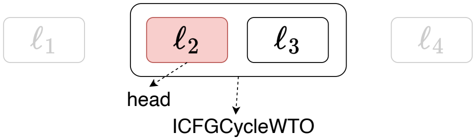
Weak Topological Order



Get ICFGCycleWTO head ℓ_2

Algorithm 12: Handle ICFG Cycle

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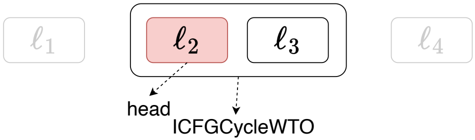
        i++;
    return;
```

Weak Topological Order



Handle ICFGCycleWTO head ℓ_2

Algorithm 12: Handle ICFG Cycle

Function `handleICFGCycle` *(cycle)*:
  ℓ := `cycle.getHead().getICFGNode()`; // cycle head ICFGNode ℓ
  `increasing` := `true`;
  `i` := 0; // analysis iteration for the loop
  while *true* **do**
    $\mathtt{as}_{\mathtt{pre}} := \sigma_\ell$; // abstract state in the last iteration
    `handleICFGNode`(ℓ);
    $\mathtt{as}_{\mathtt{cur}} := \sigma_\ell$; // abstract state in the current iteration
    if `i` $\geq$ `Options.WidenDelay()` **then**
      if `increasing` **then**
        $\sigma_\ell := \mathtt{as}_{\mathtt{pre}} \nabla \mathtt{as}_{\mathtt{cur}}$; // widening
        if $\sigma_\ell \equiv \mathtt{as}_{\mathtt{pre}}$ **then**
          `increasing` := `false`;
          continue;
      else
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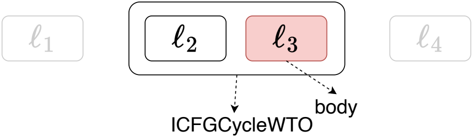
        if $\sigma_\ell \equiv \mathtt{as}_{\mathtt{pre}}$ **then**
          break;
    // continue until the widening and narrowing phases stabilize
    foreach `comp` $\in$ `cycle.getWTOComponents()` **do**
      if `comp` *is Singleton* **then**
        `handleICFGNode(comp.getICFGNode())`
      else if `comp` *is Cycle* **then**
        `handleICFGCycle(comp)`;
    `i++`;
  return;

Widening and Narrowing



Handle ICFGCycleWTO body ℓ_3

Algorithm 12: Handle ICFG Cycle

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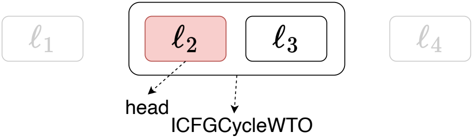
```

Note: `getIWTOcomponents` returns Cycle WTO body, i.e., ℓ_3

Widening and Narrowing



When $cur\_iter \geq Options :: WidenDelay()$

perform widening on ℓ_2

Algorithm 12: Handle ICFG Cycle

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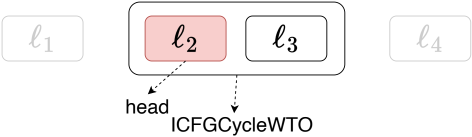
Widening and Narrowing



Widening reaches a fixpoint

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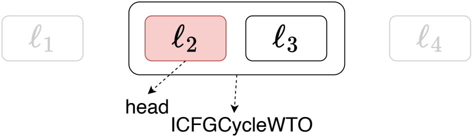
Widening and Narrowing



Perform narrowing on ℓ_2

Algorithm 12: Handle ICFG Cycle

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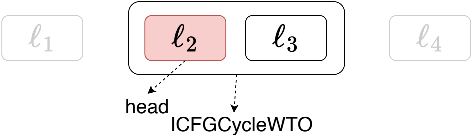
Widening and Narrowing



Narrowing reaches a fixpoint, Finish!

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```

Abstract Interpretation on SVFIR

Week 9

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Abstract Interpretation on Pointer-Free SVFIR

Interval Domain

- For simplicity, let's first consider abstract execution on a pointer-free language.
- This means there are no operations for memory allocation (like $\texttt{p} = \texttt{alloc}_o$) or for indirect memory accesses (such as `p = *q` or `*p = q`).
- Here are the pointer-free SVFSTMTs and their C-like forms:

SVFSTMT	**C-like form**
CONSSTMT	$\ell : \texttt{p} = \texttt{c}$
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$
BINARYSTMT	$\ell : \texttt{r} = \texttt{p} \otimes \texttt{q}$
PHISTMT	$\ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_n)$
SEQUENCE	$\ell_1 ; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3

Abstract Interpretation on Pointer-Free SVFIR

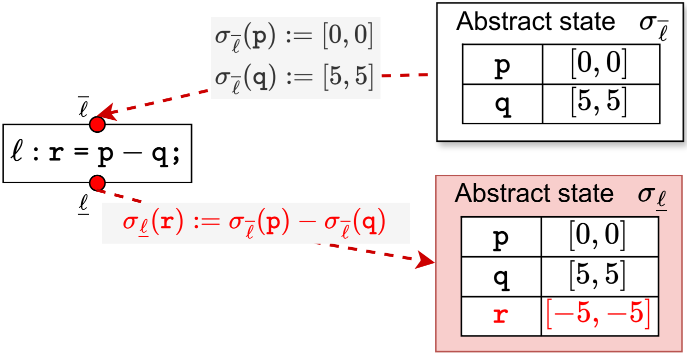
Interval Domain

Let's use the *Interval* abstract domain to update σ based on the following rules for different SVFStmt:

SVFStmt	C-like form	Abstract Execution Rule
ConsStmt	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := [\mathtt{c}, \mathtt{c}]$
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
Sequence	$\ell_1 ; \ell_2$	$\forall v \in \mathbb{V}, \sigma_{\overline{\ell_2}}(v) \sqsupseteq \sigma_{\underline{\ell_1}}(v)$
BranchStmt	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \perp$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \perp$

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$

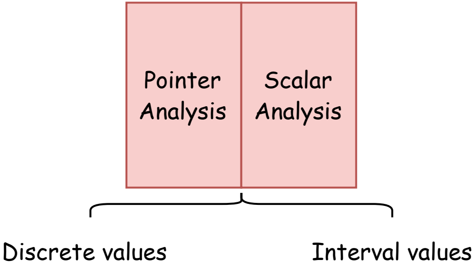


Abstract Interpretation in the Presence of Pointers

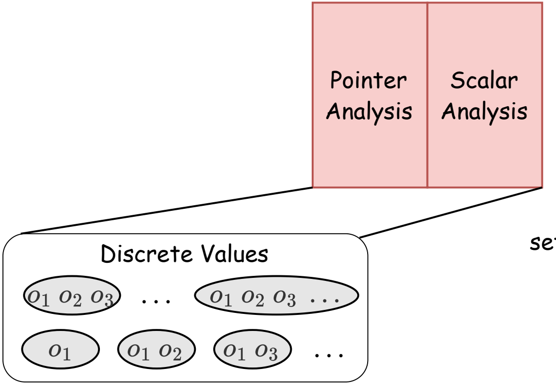
- SVFIR in the presence of pointers contains pointer-related statements including AddrStmt, GepStmt, LoadStmt and StoreStmt.
- Abstract interpretation needs to be performed on **a combined domain of intervals and abstract addresses.**

SVFStmt	**C-like form**
ConsStmt	$\ell : \mathtt{p} = \mathtt{c}$
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$
Sequence	$\ell_1 ; \ell_2$
BranchStmt	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3
AddrStmt	$\ell : \mathtt{p} = \mathtt{alloc}$
GepStmt	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$
LoadStmt	$\ell : \mathtt{p} = *\mathtt{q}$
StoreStmt	$\ell : *\mathtt{p} = \mathtt{q}$

Combined Analysis



Combined Analysis Using Discrete Values



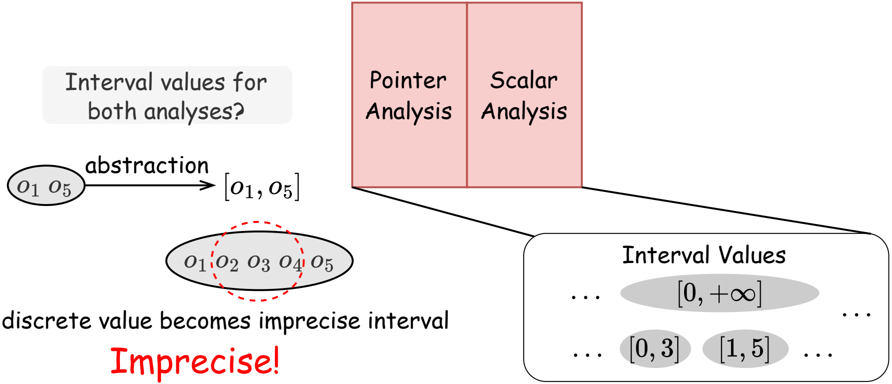
Discrete values for both analyses?

{1, 2, 3, ..., infinite numbers}

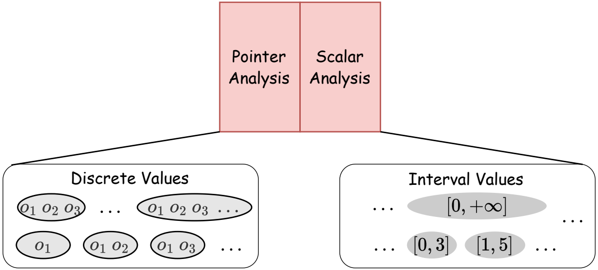
set of all possible values with inputs

Unscalable!

Combined Analysis Using Interval Values



Abstract Interpretation Over a Combined Domain

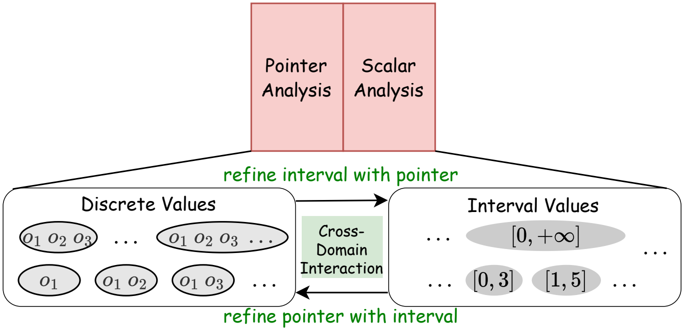



```
p = malloc(m*sizeof(int)); // p points to an array of size m
q = malloc(n*sizeof(int)); // q points to an array of size n
```

```
m = r[i];
```

- The discrete values for points-to set of `p`, `q` depend on interval values of `m` and `n`.
- The interval value of `m` depends on the pointer aliasing between `p`, `q` and `&r[i]`.
- Cyclic dependency between two domains requiring a bi-directional refinement. (variables highlighted in blue and red denote the discrete values and interval values dependent),

Abstract Interpretation Over a Combined Domain

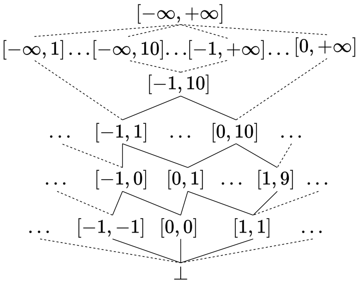


We require **a combination of interval and memory address domains** to precisely and efficiently perform abstract execution on SVFIR in the presence of pointers.

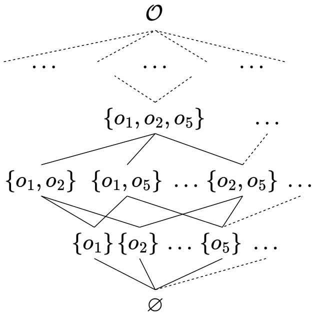
Precise Sparse Abstract Execution via Cross-Domain Interaction, ICSE 2024

Abstract Interpretation over Interval and MemAddress Domains

A Combined Domain of Intervals and Discrete Memory Addresses



Interval domain for scalar variables



MemAddress domain for discrete memory address values

SVF Program Variables (SVFVar)

Program Variables	Domain	Meanings
`SVFVar`	$\mathbb{V} = \mathbb{P} \cup \mathbb{O}$	Program Variables
`ValVar`	$\mathbb{P}$	Top-level variables (scalars and pointers)
`ObjVar`	$\mathbb{O} = \mathbb{S} \cup \mathbb{G} \cup \mathbb{H} \cup \mathbb{C}$	Memory Objects (constant data, stack, heap, global) (function objects are considered as global objects)
`FIObjVar`	$o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$	A single (base) memory object
`GepObjVar`	$o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$	*i*-th subfield/element of an (aggregate) object
`ConstantData`	$\mathbb{C}$	Constant data (e.g., numbers and strings)
Program Statement	$\ell \in \mathbb{L}$	Statements labels

Abstract Trace for the Combined Domain

- At each program point L, σ_L maps top-level variables $\mathbb{P}$ to product-domain values in $\mathit{Interval} \times \mathit{MemAddress}$ (to track the addresses or intervals).
- At each program point L, δ_L maps memory objects $\mathbb{O}$ to their abstract values.
- The pre- and post-traces store $\langle \sigma_L, \delta_L \rangle$ before and after each `ICFGNode`.

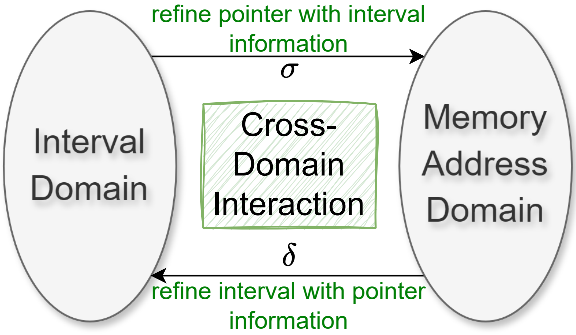
	Notation	Domain	Data Structure Implementation
Abstract trace	σ	$\mathbb{L} \times \mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$	*preAbsTrace*, *postAbsTrace*
	δ	$\mathbb{L} \times \mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$	
Abstract state	σ_L	$\mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.varToAbsVal*
	δ_L	$\mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.addrToAbsVal*
Abstract value	$\sigma_L(p)$	$\mathit{Interval} \times \mathit{MemAddress}$	*AbstractValue*
	$\delta_L(o)$		

- *Interval* is used for tracking the interval value of **scalar variables** $\mathbb{P}$.
- *MemAddress* is used for tracking the memory addresses of **memory address variables** $\mathbb{O}$.

Implementation of Abstract Trace and State in Assignment-3

- For a program point L, *AEState* consists of:
  - Top-level variable, *varToAbsVal* : $\sigma_L \in \mathbb{P} \rightarrow Interval \times MemAddress$
  - Memory object, *addrToAbsVal* : $\delta_L \in \mathbb{O} \rightarrow Interval \times MemAddress$
- The abstract trace has two maps, *preAbsTrace* and *postAbsTrace*, which maintain abstract states before and after each `ICFGNode` respectively.
  - For an `ICFGNode` ℓ, *preAbsTrace*(ℓ) retrieves the abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$, and *postAbsTrace*($\ell$) represents $\langle \sigma_{\underline{\ell}}, \delta_{\underline{\ell}} \rangle$.
  - For an abstract state $\langle \sigma, \delta \rangle$, use `as[varId]` to access σ, and use `storeValue` and `loadValue` to access δ.
  - Each variable's `AbstractValue` (e.g., `as[VarId]`) is initialized as $\perp$ in an `AbstractState` before it is assigned a new value. An uninitialized variable is assigned with $\top$ for over-approximation.
  - Each `AbstractValue` (e.g., `as[VarId]`) is a 2-element tuple consisting of an interval `as[VarId].getInterval()` and an address set `as[VarId].getAddrs()`.
  - Print out `SVFVars` and their `AbstractValues` in an `AbstractState` by invoking `as.printAbstractState()`

Abstract Trace for the Combined Domain



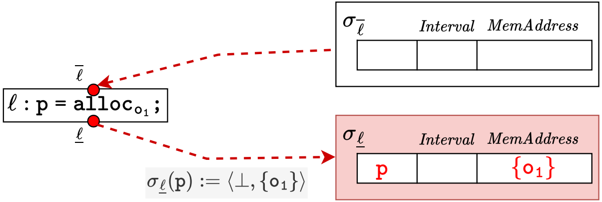
Abstract Execution Rules on SVFIR in the Presence of Pointers

Now let's use the *Interval* $\times$ *MemAddress* abstract domain to update σ and δ based on the following rules for different SVFSTMT:

SVFSTMT	**C-like form**	**Abstract Execution Rule**
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle[\mathtt{c},\mathtt{c}], \perp\rangle$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
CMPSTMT	$\ell : \mathtt{r} = \mathtt{p} \odot \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \widehat{\mathtt{cmp}}_{\odot}(\sigma_{\overline{\ell}}(p), \sigma_{\overline{\ell}}(q))$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
BRANCHSTMT	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \perp$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \perp$
SEQUENCE	$\ell_1; \ell_2$	$\delta_{\overline{\ell_2}} \sqsupseteq \delta_{\underline{\ell_1}}, \sigma_{\overline{\ell_2}} \sqsupseteq \sigma_{\underline{\ell_1}}$
ADDRSTMT	$\ell : \mathtt{p} = \mathtt{alloc}_{\mathtt{o_i}}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \perp, \{o_i\}\rangle$
GEPSTMT	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{\mathtt{o} \in \gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \perp, \{\mathtt{o.fld}_j\}\rangle$
LOADSTMT	$\ell : \mathtt{p} = *\mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \gamma(\sigma_{\overline{\ell}}(q))\}} \delta_{\overline{\ell}}(o)$
STORESTMT	$\ell : *\mathtt{p} = \mathtt{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p}))\} \sqcup \delta_{\overline{\ell}})$

Abstract Interpretation on ADDRSTMT

SVFSTMT	C-like form	Abstract Execution Rule
ADDRSTMT	$\ell : \mathtt{p} = \mathtt{alloc}_{\mathtt{o_i}}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \bot, \{o_i\} \rangle$



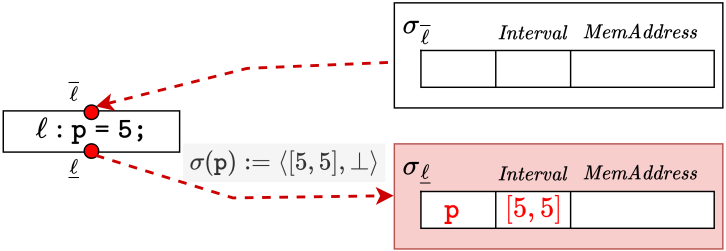
Algorithm 13: Abstract Execution Rule for ADDRSTMT

Function *updateStateOnAddr(addr)*:

```
node = addr→getICFGNode();
as = getAbsStateFromTrace(node);
initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Interpretation on ConsStmt

SVFStmt	C-like form	Abstract Execution Rule
ConsStmt	$\ell : \mathrm{p} = \mathrm{c}$	$\sigma_{\underline{\ell}}(\mathrm{p}) := \langle[\mathrm{c}, \mathrm{c}], \bot\rangle$



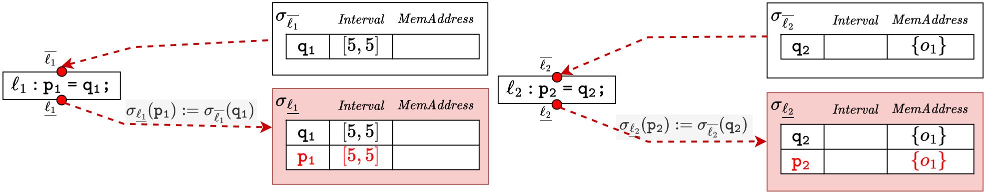
Algorithm 14: Abstract Execution Rule for ConsStmt

Function *updateStateOnAddr(addr)*:

```
node = addr→getICFGNode();
as = getAbsStateFromTrace(node);
initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Interpretation on COPYSTMT

SVFSTMT	C-like form	Abstract Execution Rule
COPYSTMT	$\ell : p = q$	$\sigma_{\underline{\ell}}(p) := \sigma_{\overline{\ell}}(q)$



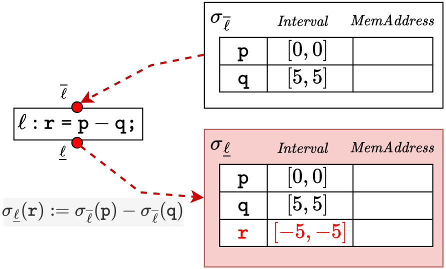
Algorithm 15: Abstract Execution Rule for COPYSTMT

Function *updateStateOnCopy(copy)*:

```
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ;
    // Assign RHS's abstract value to LHS;
```

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



Algorithm 16: Abstract Execution Rule for BINARYSTMT

```
Function updateStateOnBinary(binary):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ̄;
    // Assign the results after the binary
       operation of the two operands op0 and op1;
```

Operands `op0` and `op1` assumed to be properly initialized

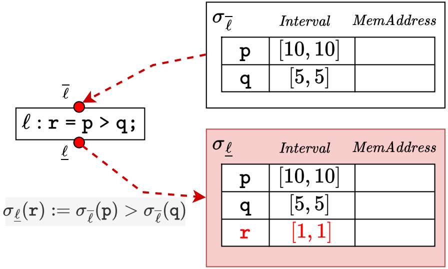
The binary operator works out the range of **all possible results** from two input intervals, use "**+**" and "**-**" as examples

$[l_1, u_1] + [l_2, u_2] = [l_1 + l_2,\ u_1 + u_2]$

$[l_1, u_1] - [l_2, u_2] = [l_1 - u_2,\ u_1 - l_2]$

Abstract Interpretation on CmpStmt

SVFStmt	C-like form	Abstract Execution Rule
CmpStmt	$\ell : \texttt{r} = \texttt{p} \odot \texttt{q}$	$\sigma_{\underline{\ell}}(r) := \widehat{\mathsf{cmp}}_{\odot}(\sigma_{\overline{\ell}}(p), \sigma_{\overline{\ell}}(q))$



Algorithm 17: Abstract Execution Rule for CmpStmt

```
Function updateStateOnCmp(cmp):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ̲;
    // Assign the results after the
    comparison operation of the two operands;
```

Operands `op0` and `op1` are assumed to be properly initialized (the Assignment-3 tests contain no reads of uninitial

Abstract Interpretation on PhiStmt

SVFStmt	C-like form	Abstract Execution Rule
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$

$\sigma_{\overline{\ell}}$

	Interval	*MemAddress*
$\mathtt{p_1}$	$[0, 0]$	
$\mathtt{p_2}$	$[5, 5]$	

$\overline{\ell}$

$\ell : \mathtt{r = phi(p_1, p_2);}$

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\mathtt{r}) := \sigma_{\overline{\ell}}(\mathtt{p_1}) \sqcup \sigma_{\overline{\ell}}(\mathtt{p_2})$

$\sigma_{\underline{\ell}}$

	Interval	*MemAddress*
$\mathtt{p_1}$	$[0, 0]$	
$\mathtt{p_2}$	$[5, 5]$	
$\mathtt{r}$	$[0, 5]$	

Algorithm 18: Abstract Execution Rule for PhiStmt

```
Function updateStateOnPhi(phi):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ̲;
    // Join the abstract values of all n
    operands retrieved from ℓ̄ or from the
    ICFGNode where each operand is defined.;
    // Assign the joined values to the result
    operand.;
    // σ_ℓ̲(r) := ⊔_{i=1}^{n} σ_ℓ̄(p_i)
```

Abstract Interpretation on GEPSTMT

SVFSTMT	C-like form	Abstract Execution Rule
GEPSTMT	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \bot, \{\mathtt{o.fld}_j\} \rangle$

Note: `getGepObjAddrs` computes abstract target addresses without explicitly enumerating an unbounded concretization; unknown indices are handled conservatively by the Assignment-3 memory model.

$\overline{\ell}$

`ℓ : p = &(q[i]);`

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \bot, \{\mathtt{o}_1.\mathtt{fld}_1\} \rangle \sqcup \langle \bot, \{\mathtt{o}_1.\mathtt{fld}_2\} \rangle$

$\sigma_{\overline{\ell}}$	*Interval*	*MemAddress*
q		$\{\mathtt{o}_1\}$
i	$[1, 2]$	

$\sigma_{\underline{\ell}}$	*Interval*	*MemAddress*
q		$\{\mathtt{o}_1\}$
i	$[1, 2]$	
p		$\{\mathtt{o}_1.\mathtt{fld}_1, \mathtt{o}_1.\mathtt{fld}_2\}$

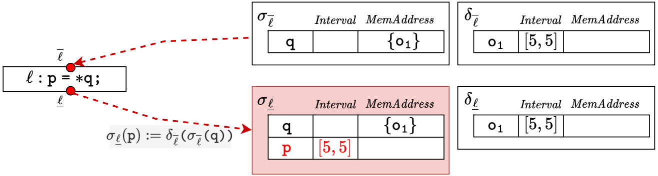
Algorithm 19: Abstract Execution Rule for GEPSTMT

Function *updateStateOnGep(gep)*:
```
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ;
    // Retrieve the field index or array index i
    given as.getElementIndex(gep);
    // Retrieve the memory address value via
    as.getGepObjAddrs(rhs, i) and assign it to
    LHS
```

Abstract Interpretation on LoadStmt

SVFStmt	C-like form	Abstract Execution Rule
LoadStmt	ℓ : p = *q	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \gamma(\sigma_{\overline{\ell}}(q))\}} \delta_{\overline{\ell}}(o)$

Note: a load joins the abstract values of all possible pointees.



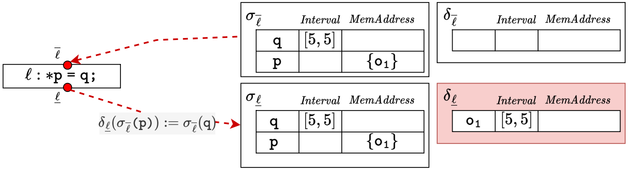
Algorithm 20: Abstract Execution Rule for LoadStmt

```
Function updateStateOnLoad(load):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ;
    // Load the value from RHS via
       as.loadValue(rhs) and assign it to LHS;
```

Abstract Interpretation on STORESTMT

SVFSTMT	C-like form	Abstract Execution Rule
STORESTMT	$\ell : *\mathrm{p} = \mathrm{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathrm{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathrm{p}))\} \sqcup \delta_{\underline{\ell}})$

Note: the rule performs a weak update. A strong update is possible only when the target is known to be exactly one singleton abstract object.



Algorithm 21: Abstract Execution Rule for STORESTMT

Function *updateStateOnStore(store)*:
```
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ;
    // Store RHS value to LHS via as.storeValue;
```

An Example: Abstract Trace σ for Top-level Variables

```c
extern void assert(int);

int main(){
    int a = 0;
    while(a < 10) {
        a++;
    }
    assert(a == 10);
    return 0;
}
```

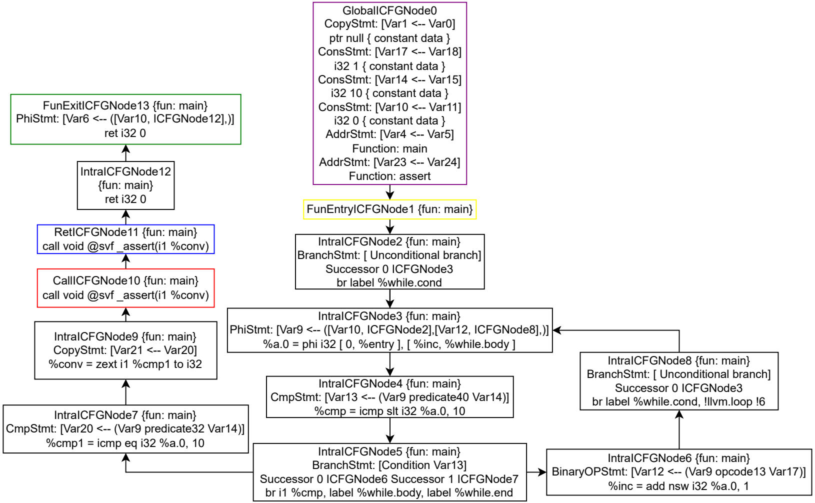
Source Code

Compile to LLVM IR

```llvm
define dso_local i32 @main() {
entry:
  br label %while.cond
while.cond:
  %a.0 = phi i32 [ 0, %entry ], [ %inc, %while.body ]
  %cmp = icmp slt i32 %a.0, 10
  br i1 %cmp, label %while.body, label %while.end
while.body:
  %inc = add nsw i32 %a.0, 1
  br label %while.cond,
while.end:
  %cmp1 = icmp eq i32 %a.0, 10
  %conv = zext i1 %cmp1 to i32
  call void @assert(i32 noundef %conv)
  ret i32 0
}
```

LLVM IR

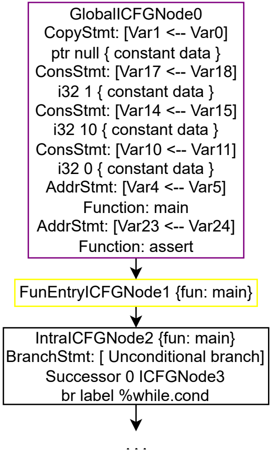
An Example: Abstract Trace σ for Top-level Variables



ICFG

An Example: Abstract Trace σ for Top-level Variables

Before Entering Loop



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Algorithm 22: Abstract execution guided by WTO

Function `handleStatement`(ℓ)**:**
- $tmpAS := preAbsTrace[\ell]$;
- **if** ℓ *is* ConsStmt *or* AddrStmt **then**
  - `updateStateOnAddr`(ℓ);
- **else if** ℓ *is* CopyStmt **then**
  - `updateStateOnCopy`(ℓ);
- $\ldots$;

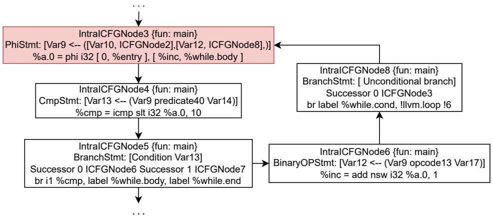
$postAbsTrace[ICFGNode0].varToAbsVal$:

SVFVar	AbstractValue$\langle Interval, MemAddress\rangle$
$Var0$	$\langle \bot, \{0x7f00\}\rangle$
$Var1$	$\langle \bot, \{0x7f00\}\rangle$
$Var18$	$\langle [1,1], \bot\rangle$
$Var17$	$\langle [1,1], \bot\rangle$
$Var14$	$\langle [10,10], \bot\rangle$
$Var15$	$\langle [10,10], \bot\rangle$
$Var10$	$\langle [0,0], \bot\rangle$
$Var11$	$\langle [0,0], \bot\rangle$
	...

Print out the table via `as.printAbstractState()`. The `AbstractValue` can **either be an interval or addresses**, but not both!

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

postAbsTrace[*ICFGNode3*].*varToAbsVal* :

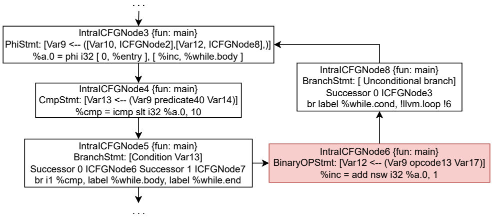
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var10	$\langle [0,0], \perp \rangle$
Var9	$\langle [0,0], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

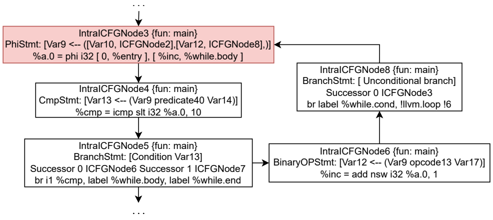
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var10	$\langle [0,0], \bot \rangle$
Var9	$\langle [0,0], \bot \rangle$
Var12	$\langle [1,1], \bot \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0, 1], \perp \rangle$
Var12	$\langle [1, 1], \perp \rangle$
	...

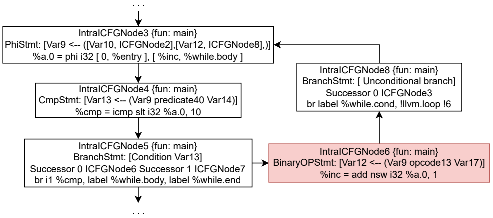
Refer back to the example in Week-9.

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

postAbsTrace[*ICFGNode6*].*varToAbsVal* :

SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0,1], \perp \rangle$
Var12	$\langle [1,2], \perp \rangle$
	...

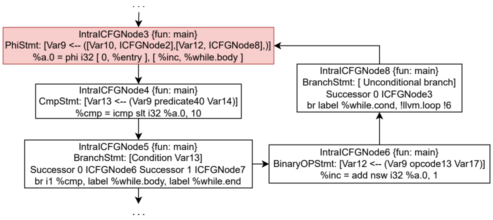
Refer back to the example in Week-9.

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

postAbsTrace[*ICFGNode3*].*varToAbsVal* :

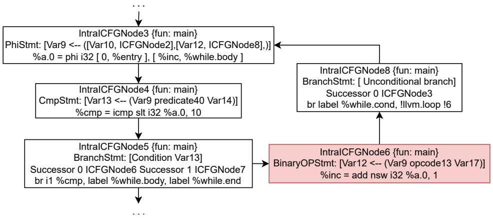
SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
	...
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, +\infty], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

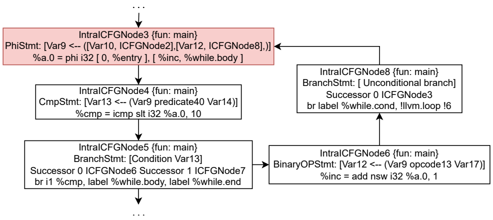
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0, 9], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase Fixed Point



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

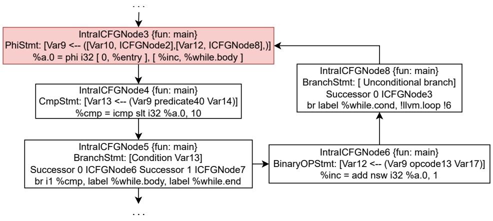
SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
	...
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, +\infty], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

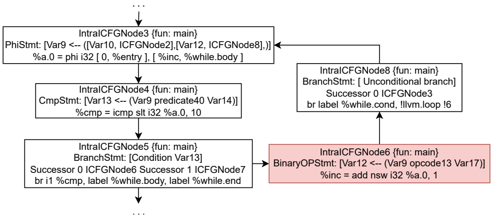
SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
	. . .
Var9	$\langle[0, 10], \perp\rangle$
Var12	$\langle[1, 10], \perp\rangle$
	. . .

Algorithm 12: Handle ICFG Cycle

Function handleICFGCycle *(cycle)*:
- ℓ := cycle.getHead().getICFGNode(); // cycle head ICFGNode ℓ
- increasing := true;
- i := 0; // analysis iteration for the loop
- **while** *true* **do**
  - $as_{pre} := \sigma_\ell$; // abstract state in the last iteration
  - handleICFGNode(ℓ);
  - $as_{cur} := \sigma_\ell$; // abstract state in the current iteration
  - **if** i $\geq$ Options.WidenDelay() **then**
    - **if** increasing **then**
      - $\sigma_\ell := as_{pre} \nabla as_{cur}$; // widening
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - increasing := false;
        - **continue**;
    - **else**
      - $\sigma_\ell := as_{pre} \Delta as_{cur}$; // narrowing
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - **break**;
  - // continue until the widening and narrowing phases stabilize
  - **foreach** comp $\in$ cycle.getWTOComponents() **do**
    - **if** comp *is Singleton* **then**
      - handleICFGNode(comp.getICFGNode())
    - **else if** comp *is Cycle* **then**
      - handleICFGCycle(comp);
  - i++;
- **return**;

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

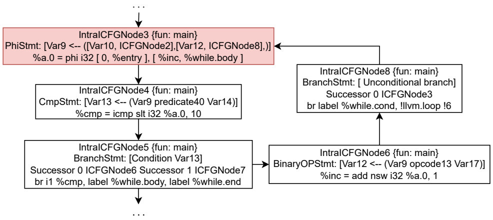
SVFVar	$\langle Interval, MemAddress\rangle$
	...
Var9	$\langle [0, 9], \perp\rangle$
Var12	$\langle [1, 10], \perp\rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // continue until the widening and narrowing phases stabilize
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase Fixed Point



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

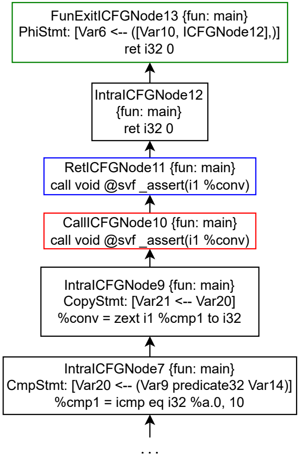
SVFVar	$\langle Interval, MemAddress \rangle$
	...
Var9	$\langle [0, 10], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

Function handleICFGCycle *(cycle)*:
  ℓ := cycle.getHead().getICFGNode(); // cycle head ICFGNode ℓ
  increasing := true;
  i := 0; // analysis iteration for the loop
  while *true* **do**
    $as_{pre} := \sigma_\ell$; // abstract state in the last iteration
    handleICFGNode(ℓ);
    $as_{cur} := \sigma_\ell$; // abstract state in the current iteration
    if i $\geq$ Options.WidenDelay() **then**
      if increasing **then**
        $\sigma_\ell := as_{pre} \nabla as_{cur}$; // widening
        if $\sigma_\ell \equiv as_{pre}$ **then**
          increasing := false;
          continue;
      else
        $\sigma_\ell := as_{pre} \Delta as_{cur}$; // narrowing
        if $\sigma_\ell \equiv as_{pre}$ **then**
          break;
    // continue until the widening and narrowing phases stabilize
    foreach comp $\in$ cycle.getWTOComponents() **do**
      if comp *is Singleton* **then**
        handleICFGNode(comp.getICFGNode())
      else if comp *is Cycle* **then**
        handleICFGCycle(comp);
    i++;
  return;

An Example: Abstract Trace σ for Top-level Variables

After Exiting Loop



ICFG

Algorithm 13: Abstract execution guided by WTO

Function `handleStatement`(ℓ):
- $tmpAS := preAbsTrace[\ell]$;
- **if** ℓ *is* CmpStmt **then**
  - `updateStateOnCmp`(ℓ);
- $\ldots$;

$postAbsTrace[ICFGNode7].varToAbsVal$:

`SVFVar`	$\langle Interval, MemAddress \rangle$
	$\ldots$
$Var9$	$\langle [10, 10], \perp \rangle$
$Var20$	$\langle [1, 1], \perp \rangle$
	$\ldots$