Recursion -- Solving Problems with your Mind...

And some C code...

“Isn’t everything solving problems with your mind?”

~Marc Chee, 2021

What are we learning about today

1. A new way of thinking about solving problems.
2. A way to write shorter but elegant programs.

How have we learned to solve a problem so far?

Problem: Take a linked list of positive integers, and find the largest number.

How have we learned to solve a problem so far?

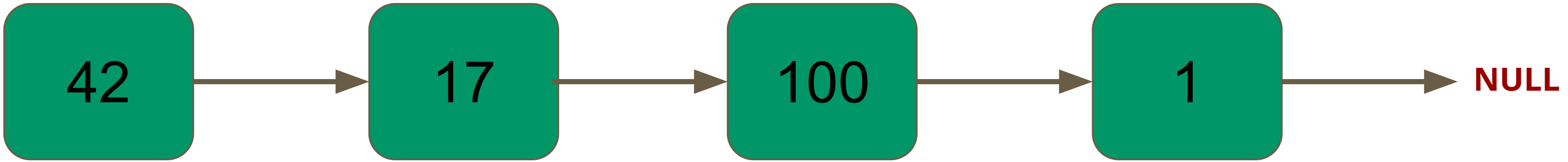
Problem: Take a linked list of positive integers, and find the largest number.

Solution:

- Initialize a variable called `max` to something really small (-1)
- Loop through each node of the linked list.
- If `curr->value` is bigger than `max`, update `max`

This is a perfectly fine way to solve this problem!

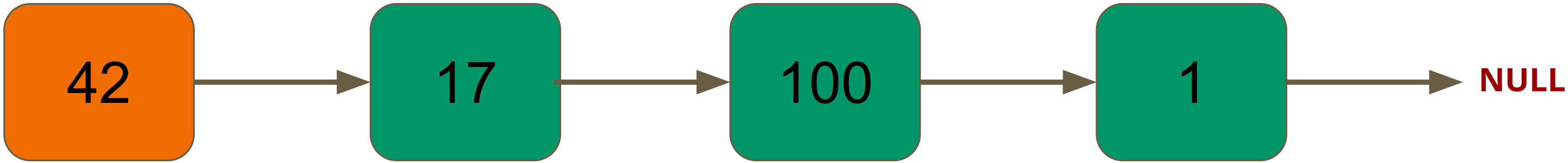
max = -1




```
// Find the maximum value in the linked list
int find_max(struct node *head) {
    int max = -1;
    struct node *curr = head;

    while (curr != NULL) {
        if (max < curr->value) {
            max = curr->value;
        }
        curr = curr->next;
    }
    return max;
}
```

max = 42 (was -1)

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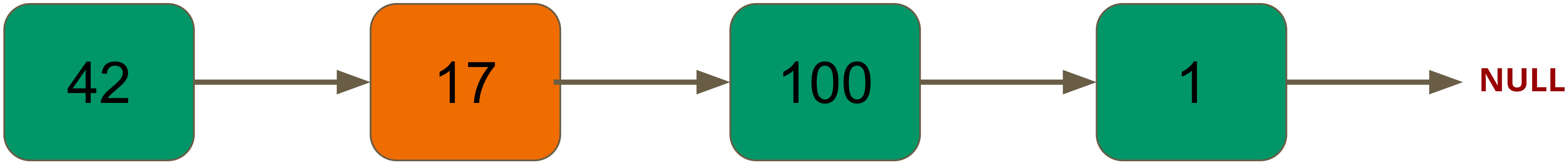
    return max;
}
```

max = 42

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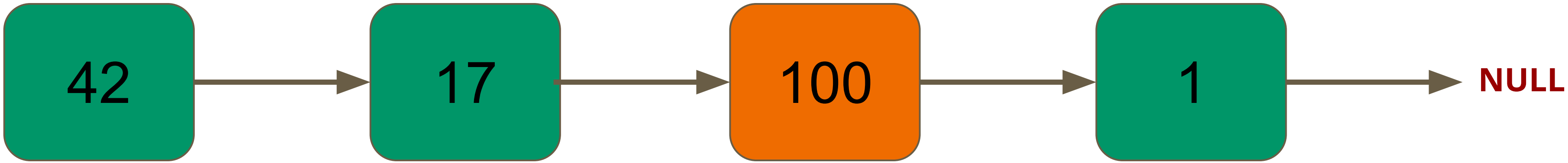
    while (curr != NULL) {
        if (max < curr->value) {
            max = curr->value;
        }
        curr = curr->next;
    }
    return max;
}
```

max = 100 (was 42)

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    struct node *curr = head;

    while (curr != NULL) {
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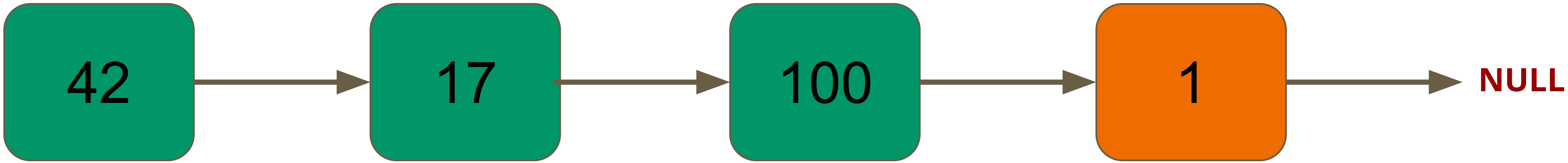
        }
        curr = curr->next;
    }
    return max;
}
```

max = 100

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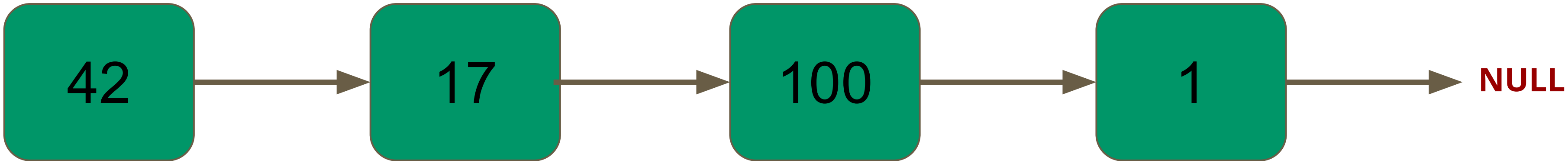
        if (max < curr->value) {
            max = curr->value;
        }
        curr = curr->next;
    }
    return max;
}
```

🎉 🎉 🎉 max = 100 🎉 🎉 🎉

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    while (curr != NULL) {
        if (max < curr->value) {
            max = curr->value;
        }
        curr = curr->next;
    }
    return max;
}
```

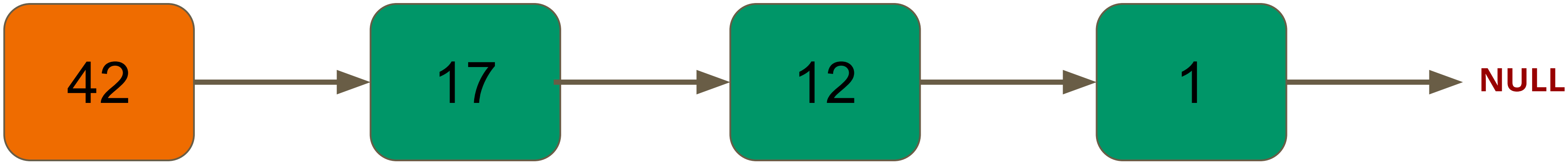
But now... a new way!

Problem: Take a linked list of positive integers, and find the largest number.

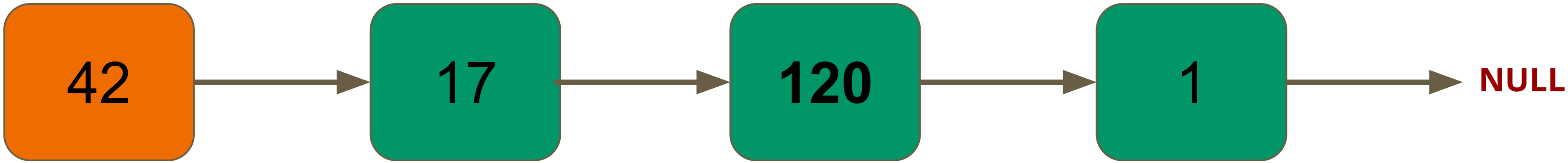
Observation: If we have a linked list with at least one node in it, there are two possibilities:

1) The largest number is the first number.
2) The largest number is something **other** than the first number.

Possibility #1: Largest number is the current one



Possibility #2: Largest number is in the rest of the list



So what?

Problem: Take a linked list of positive integers, and find the largest number.

Solution:

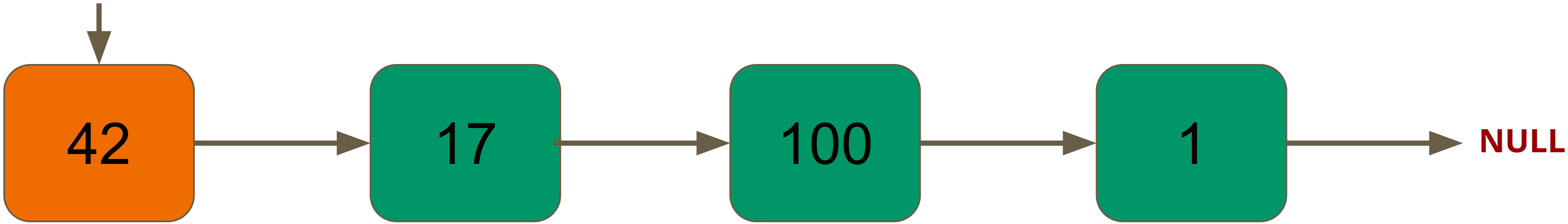
- Find the maximum number of everything *after* the current node
- Either:
  - Everything in the rest of the list is *smaller* than the current element.
  - Or, something in the rest of the list is **bigger** than the current element.

Functions call themselves?!?!?!

```
// Find the maximum value in the linked list
int find_max(struct node *head) {
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```

maximum_so_far = **the max of the rest of the list**




```c
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int find_max(struct node *head) {
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```
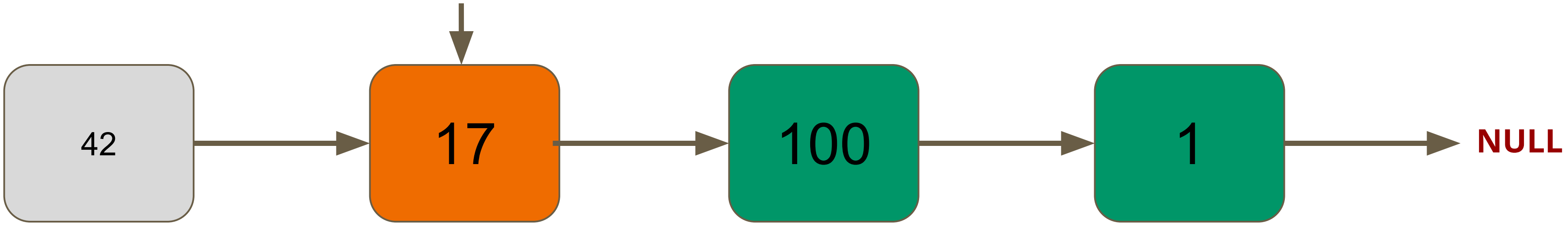

maximum_so_far = **the max of the rest of the list**

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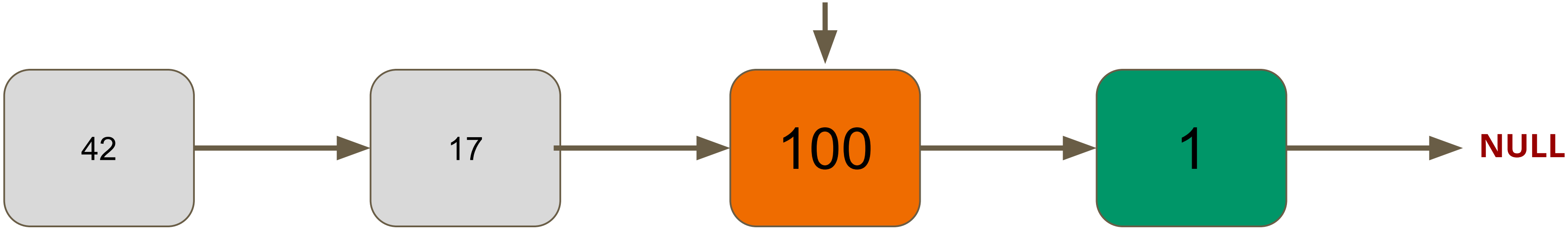
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```

maximum_so_far = **the max of the rest of the list**




```
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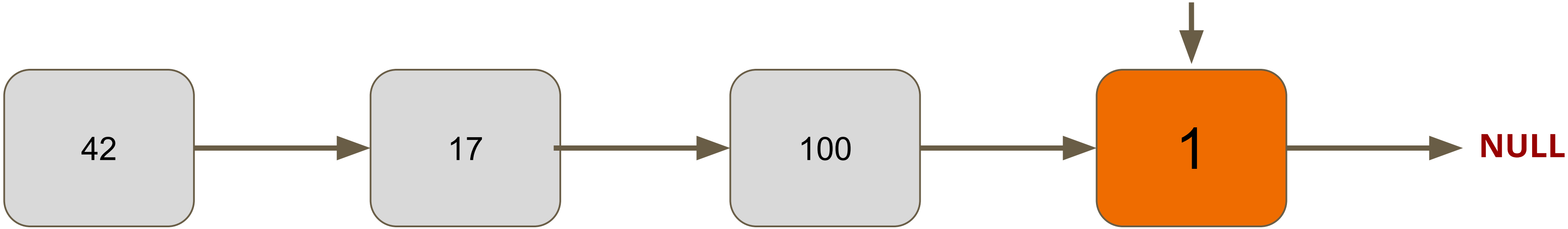
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```

maximum_so_far = **the max of the rest of the list**




```
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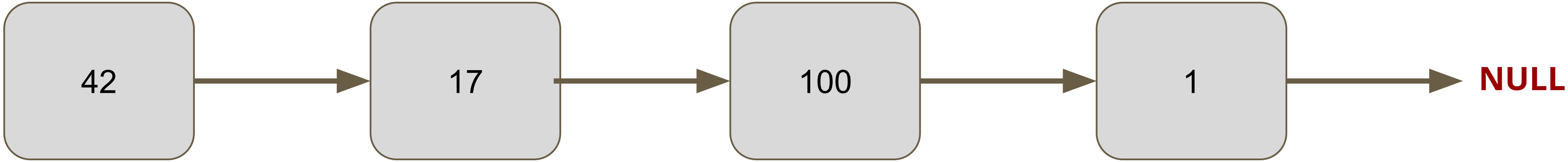
int find_max(struct node *head) {
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```

max = ?????

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int find_max(struct node *head) {
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }

    return maximum_so_far;
}
```

Uh Oh...

```
/tmp/starter_code.c:18:41: runtime error - accessing a field via a NULL pointer

dcc explanation: You are using a pointer which is NULL
  A common error is  using p->field when p == NULL.

Execution stopped in find_max(head=NULL) in /tmp/starter_code.c at line 18:

int find_max(struct node *head) {
--> int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;

Values when execution stopped:

head = NULL
maximum_so_far = <uninitialized value>

Function Call Traceback
find_max(head=NULL) called at line 37 of /tmp/starter_code.c
main()
```

We should have known...

```
starter_code.c:17:33: warning: all paths through this function will call itself [-Winfinite-recursion]
int find_max(struct node *head) {
                                ^
```

One problem...

What do we do when we get to the end of the list? Currently we have “infinite recursion”.

So we need to include a special case: “what to do when we have no elements left”. We call this the *base case*.

Solution: Return a negative number (-1) if given an empty list. Since our list always contains positive integers, it’s smaller than anything else.

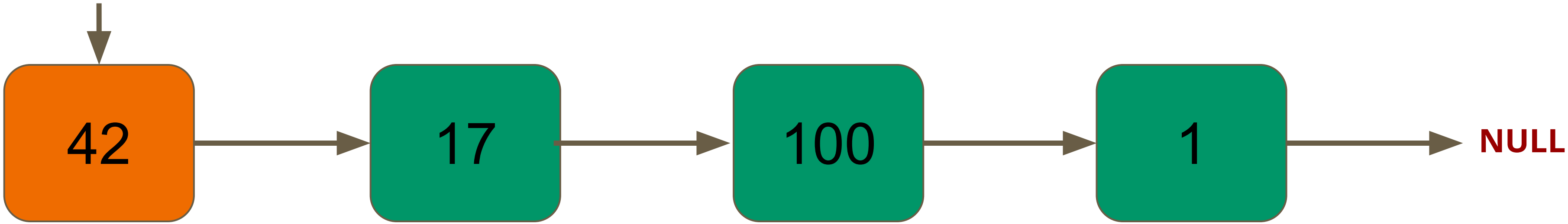
Solution, Attempt 2 [this time it'll work [hopefully]]

Problem: Take a linked list, and find the largest number.

Solution:

- Find the maximum value in nodes *after* the current node
- Either:
    - *We have an empty list, in which case return -1*
    - Everything in the rest of the list is *smaller* than the current element.
    - Or, something in the rest of the list is **bigger** than the current element.

maximum_so_far = **the max of the rest of the list**




```c
#define NO_MAXIMUM -1

// Find the maximum value in the linked list
int find_max(struct node *head) {
    if (head == NULL)  {
        return NO_MAXIMUM;
    }
    int maximum_so_far = find_max(head->next);
    if (maximum_so_far < head->value) {
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        maximum_so_far = head->value;
    }
    return maximum_so_far;
}
```
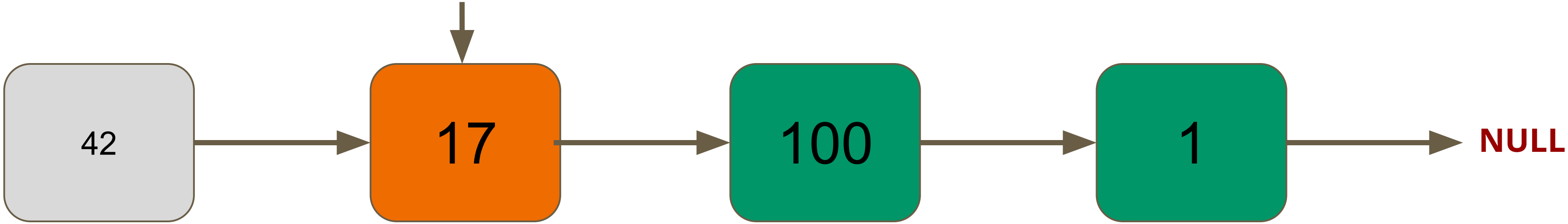

maximum_so_far = **the max of the rest of the list**

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    }
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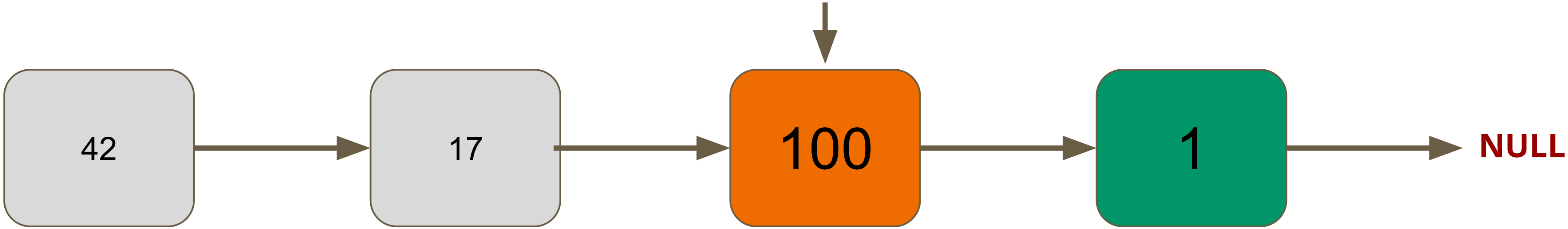
}
```

maximum_so_far = **the max of the rest of the list**

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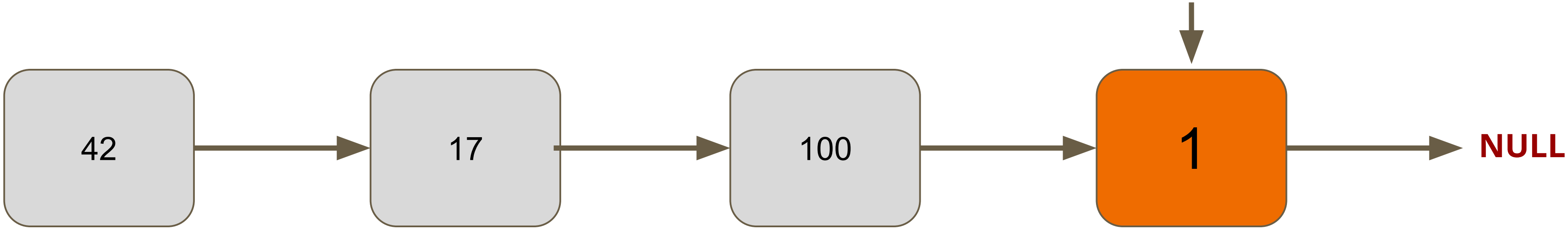


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    }
    return maximum_so_far;
}
```
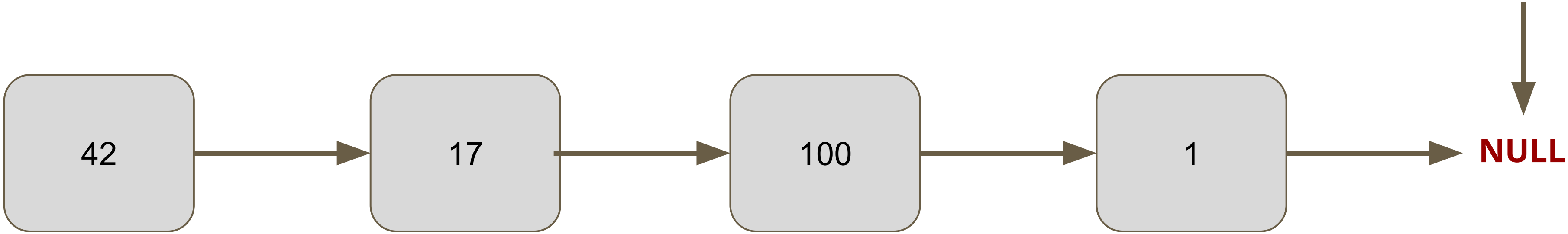

maximum_so_far = **-1**

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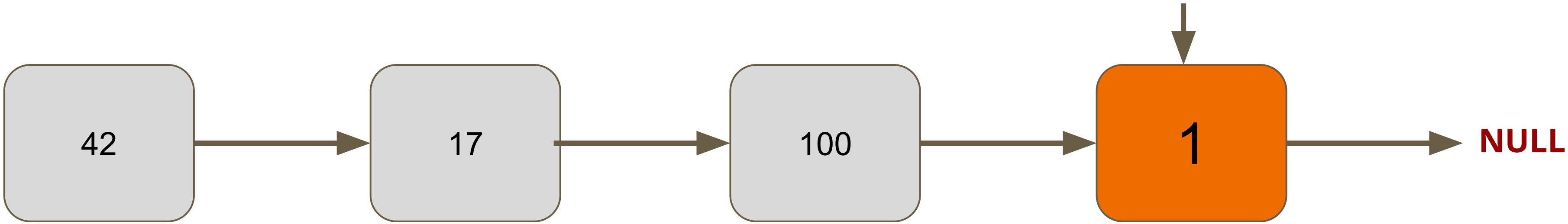
    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }
    return maximum_so_far;
}
```

maximum_so_far = 1 **or** -1

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    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }
    return maximum_so_far;
}
```
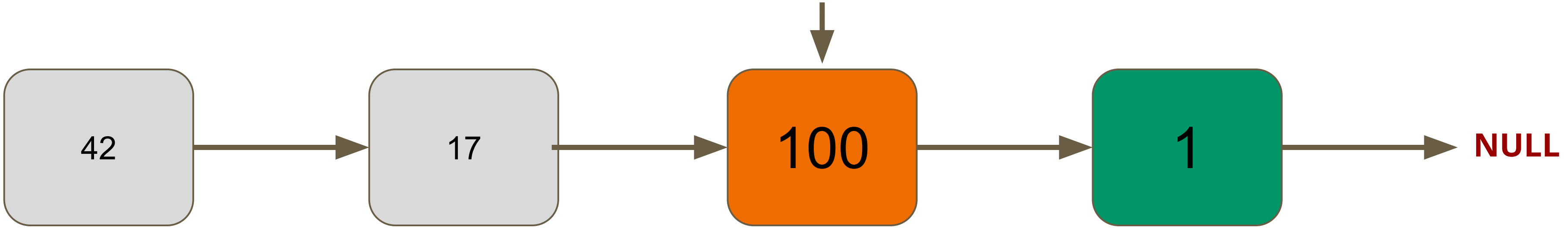

maximum_so_far = **100** **or** 1

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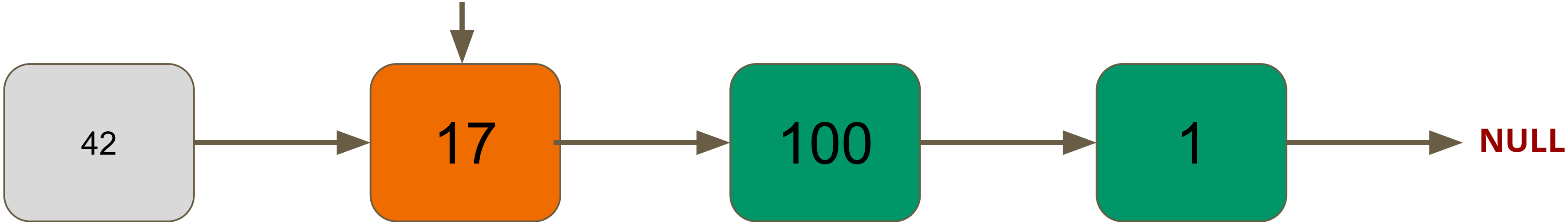
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }
    return maximum_so_far;
}
```

maximum_so_far = 17 **or** 100

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    if (maximum_so_far < head->value) {
        // we have found a new node that's bigger
        maximum_so_far = head->value;
    }
    return maximum_so_far;
}
```
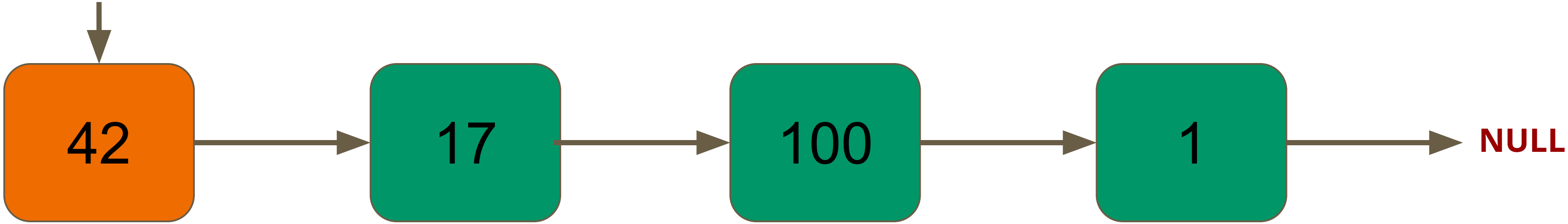

maximum_so_far = 42 **or** 100

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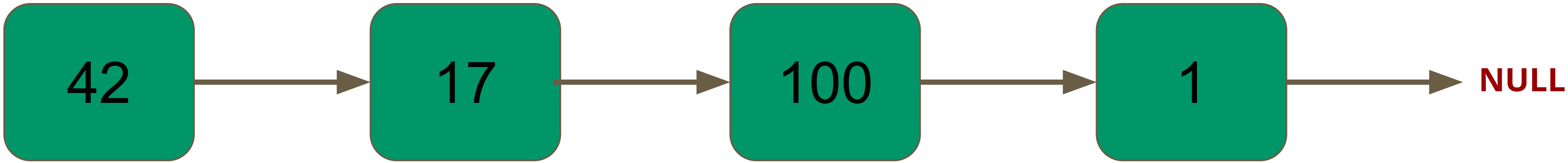
}
```

🎉 🎉 🎉 maximum_so_far = 100 🎉 🎉 🎉

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    }
    return maximum_so_far;
}
```
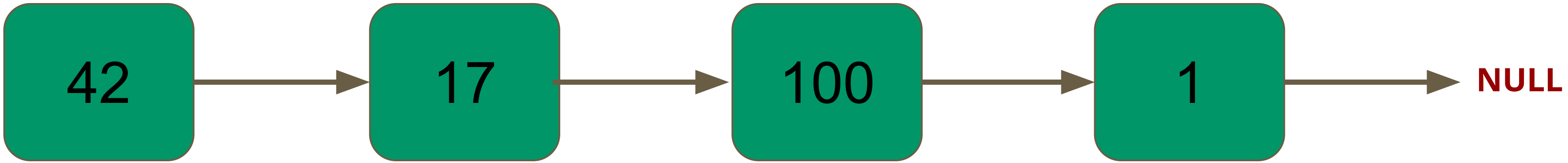

Let’s take a look at some real code...

Why is Recursion different from a While Loop

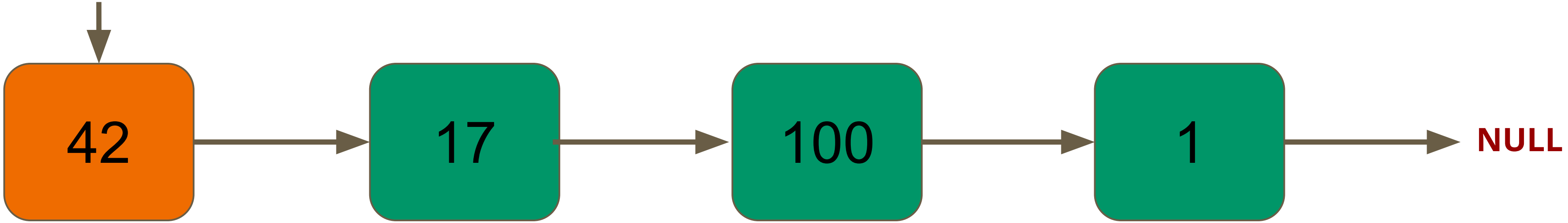
A normal while loop solves your problem by a series of steps that ends up with the right answer.

Recursion solves your problem by solving a stack of smaller problems. Then, it deals with the trivially easy case.



main()

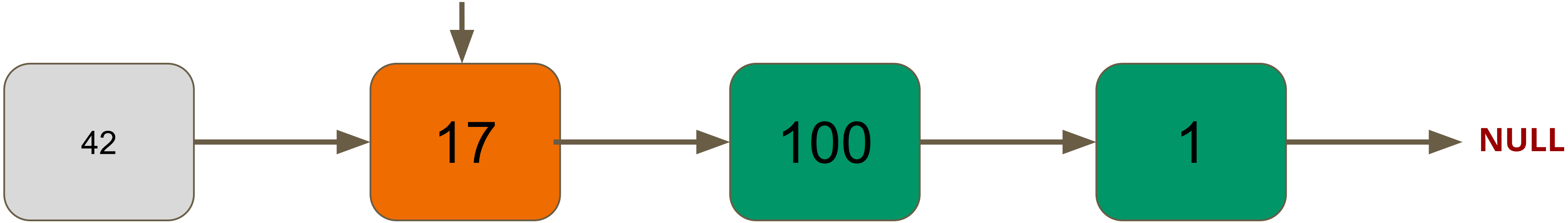
maximum_so_far = **the max of the rest of the list**



find_max() [42 is at head of list]

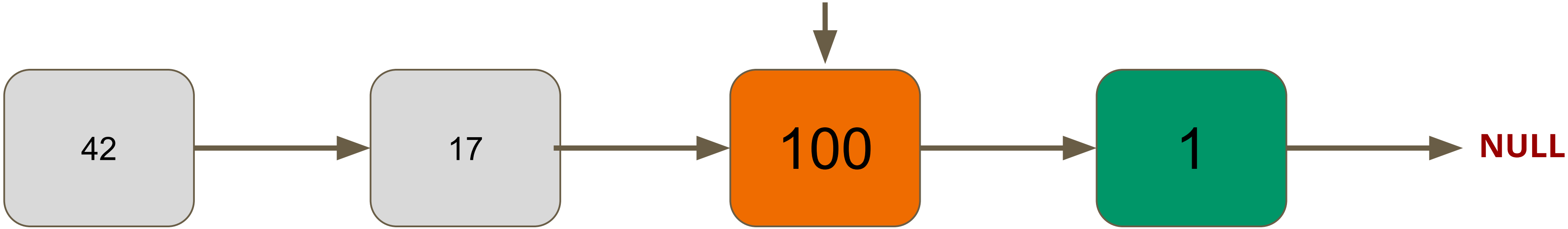
main()

maximum_so_far = **the max of the rest of the list**



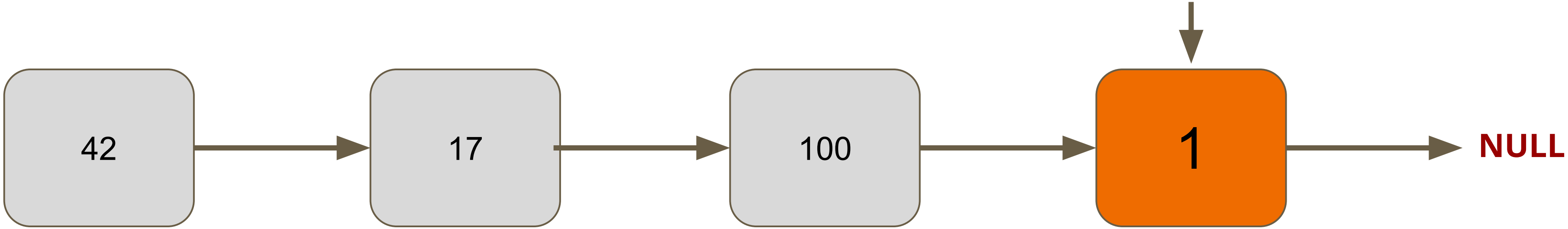
find_max() [17 is at head of list]
find_max() [42 is at head of list]
main()

maximum_so_far = **the max of the rest of the list**



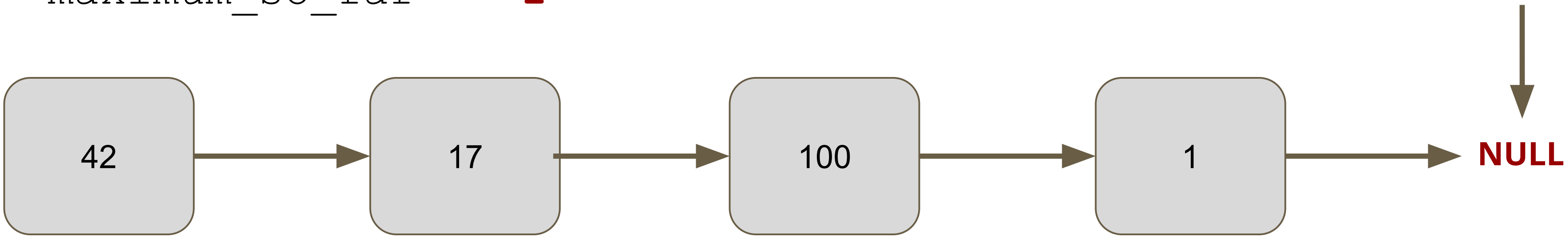
find_max() [100 is at head of list]
find_max() [17 is at head of list]
find_max() [42 is at head of list]
main()

maximum_so_far = **the max of the rest of the list**



find_max() [1 is at head of list]
find_max() [100 is at head of list]
find_max() [17 is at head of list]
find_max() [42 is at head of list]
main()

maximum_so_far = **-1**



find_max() [NULL is at head of list]

find_max() [1 is at head of list]

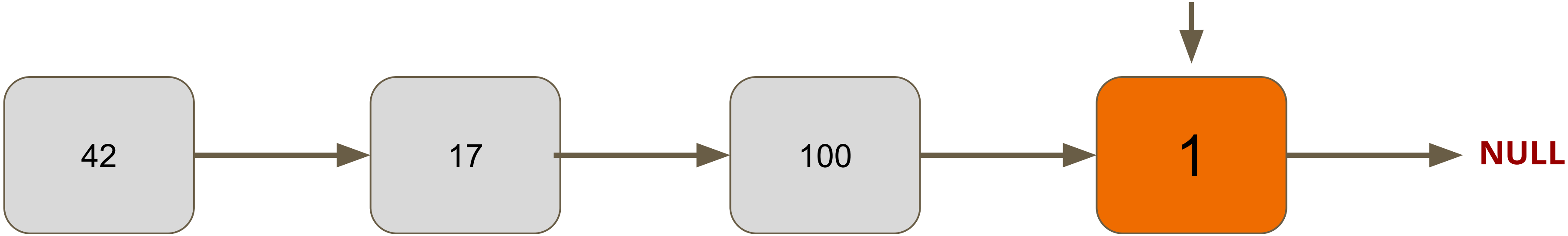
find_max() [100 is at head of list]

find_max() [17 is at head of list]

find_max() [42 is at head of list]

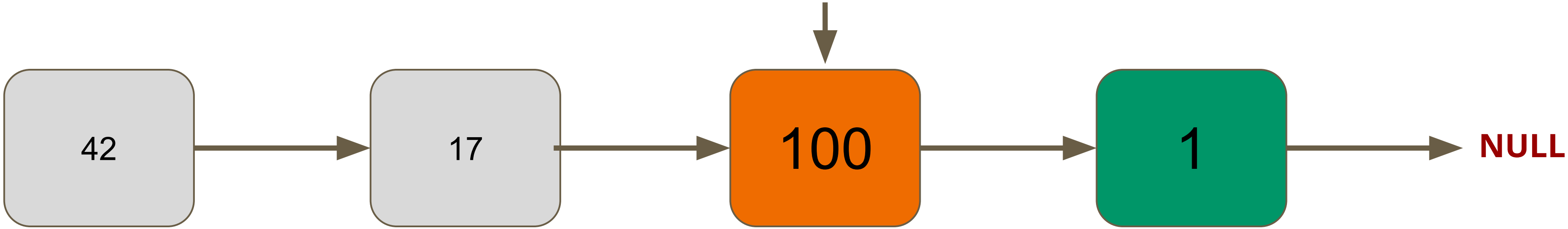
main()

`maximum_so_far = 1 or -1`



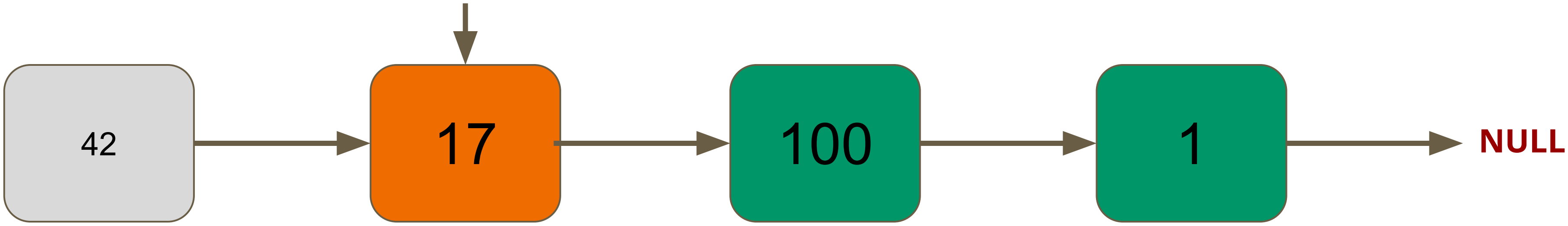
find_max() [1 is at head of list]
find_max() [100 is at head of list]
find_max() [17 is at head of list]
find_max() [42 is at head of list]
main()

maximum_so_far = 100 **or** 1



find_max() [100 is at head of list]
find_max() [17 is at head of list]
find_max() [42 is at head of list]
main()

maximum_so_far = 17 **or** 100

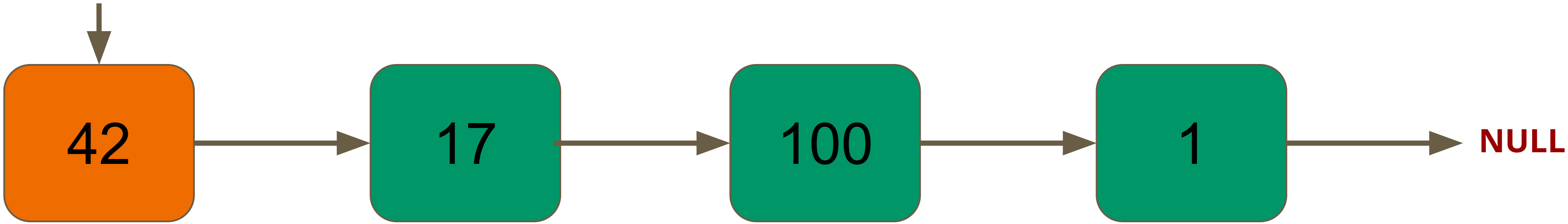


find_max() [17 is at head of list]

find_max() [42 is at head of list]

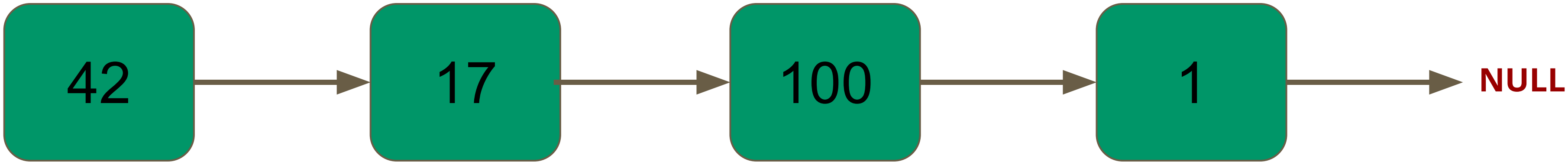
main()

```
maximum_so_far = 42 or 100
```




```
find_max()  [42 is at head of list]
main()
```

🎉 🎉 🎉 maximum_so_far = 100 🎉 🎉 🎉



main()

Break Time...

Why is recursion so cool?

- Functions are incredibly powerful - in fact, they can do anything a w
- Turns out, we don’t even need variables at all -- we can just use functions and never modify our variables
- “Functional Programming” is a different style of programming where you never assign to variables; and *everything* is a function.
- The most common example of functional programming is...

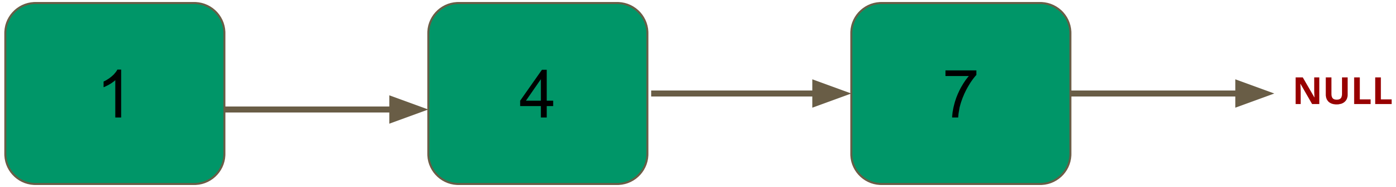
X

Benefits of Recursion -- Printing Forwards and Backwards

Benefits of Recursion -- No Need for a Prev Pointer

Let's look at a more interesting example...

Let's say we have a list; can we figure out **every** way to add it it's nodes together?



0 + 0 + 0 = 0	**1** + 0 + 0 = 1	0 + **4** + 0 = 4	**1** + **4** + 0 = 5	0 + 0 + **7** = 7	**1** + **4** + **7** = 11

What did we learn today?

1. A new way of thinking about solving problems.
    a. Instead of solving big problems, solve smaller, easier problems and build them up!
2. A way to write shorter but elegant programs.
    a. Recursion can often be neater than using while loops, at the cost of requiring more time to think through.