

## COMP9517: Computer Vision

Motion

## Introduction

- Adding the time dimension to the image formation



## Introduction

- A changing scene may be observed via a sequence of images



## Introduction

- Changes in an image sequence provide features for
- detecting objects that are moving
- computing trajectories of moving objects
- performing motion analysis of moving objects
- recognising objects based on their behaviours
- computing the motion of the viewer in the world
- detecting and recognising activities in a scene


## Applications

- Motion-based recognition
- human identification based on gait, automatic object detection
- Automated surveillance
- monitoring a scene to detect suspicious activities or unlikely events
- Video indexing
- automatic annotation and retrieval of videos in multimedia databases
- Human-computer interaction
- gesture recognition, eye gaze tracking for data input to computers
- Traffic monitoring
- real-time gathering of traffic statistics to direct traffic flow
- Vehicle navigation
- video-based path planning and obstacle avoidance capabilities


## Scenarios

- Still camera

Constant background with

- single moving object

- Moving camera

Relatively constant scene with

- coherent scene motion
- single moving object
- multiple moving objects



## Topics

- Change detection

Using image subtraction to detect changes in scenes

- Sparse motion estimation

Using template matching to estimate local displacements

- Dense motion estimation

Using optical flow to compute a dense motion vector field

## Change Detection

## Change Detection

- Detecting an object moving across a constant background
- The forward and rear edges of the object advance only a few pixels per frame

- By subtracting the image $I_{t}$ from the previous image $I_{t-1}$ the edges should be evident as the only pixels significantly different from zero


## Image Subtraction

Step: Derive a background image from a set of video frames at the beginning of the video sequence


PETS 2009
Benchmark

## Image Subtraction

Step: Subtract the background image from each subsequent frame to create a difference image


## Image Subtraction

Step: Threshold and enhance the difference image to fuse neighbouring regions and remove noise


## Change Detection

- Image subtraction algorithm
- Input: images $I_{t}$ and $I_{t-\Delta t}$ (or a model image)
- Input: an intensity threshold $\tau$
- Output: a binary image $\mathrm{I}_{\text {out }}$
- Output: a set of bounding boxes B

1. For all pixels $[r, c]$ in the input images,
set $I_{\text {out }}[r, c]=1$ if $\left(\left|I_{t}[r, c]-I_{t-\Delta t}[r, c]\right|>\tau\right)$
set $I_{\text {out }}[r, c]=0$ otherwise
2. Perform connected components extraction on $I_{\text {out }}$
3. Remove small regions in $\mathrm{I}_{\text {out }}$ assuming they are noise
4. Perform a closing of $\mathrm{I}_{\text {out }}$ using a small disk to fuse neighbouring regions
5. Compute the bounding boxes of all remaining regions of changed pixels
6. Return $\mathrm{I}_{\text {out }}[\mathrm{r}, \mathrm{c}]$ and the bounding boxes B of regions of changed pixels

## Sparse Motion Estimation

## Motion Vector

- A motion field is a 2D array of 2D vectors representing the motion of 3D scene points
- A motion vector in the image represents the displacement of the image of a moving 3D point
- Tail at time $t$ and head at time $t+\Delta t$
- Instantaneous velocity estimate at time t


Zoom out


Zoom in


Pan Left

## Sparse Motion Estimation

- A sparse motion field can be computed by identifying pairs of points that correspond in two images taken at times $t$ and $t+\Delta t$
- Assumption: intensities of interesting points and their neighbours remain nearly constant over time
- Two steps:
- Detect interesting points at t
- Search corresponding points at $\mathrm{t}+\Delta \mathrm{t}$



## Sparse Motion Estimation

- Detect interesting points
- Image filters
- Canny edge detector
- Kirsch edge operator
- Harris corner detector
- SUSAN corner detector
- Frei-Chen ripple operator
- ...
- Interest operator
- Computes intensity variance in the vertical, horizontal and diagonal directions
- Interest point if the minimum of these four variances exceeds a threshold


## Detect Interesting Points

```
Procedure detect_interesting_points(I,V,w,t) {
    for (r = 0 to MaxRow - 1)
        for (c = 0 to MaxCol - 1)
            if (I[r,c] is a border pixel) break;
            else if (interest_operator(I,r,c,w) >= t)
            add (r,c) to set V;
}
```

Procedure interest_operator (l,r,c,w) \{
$\mathrm{v} 1=$ variance of intensity of horizontal pixels $\mathrm{I}[\mathrm{r}, \mathrm{c}-\mathrm{w}] . . . \mid[\mathrm{r}, \mathrm{c}+\mathrm{w}]$;
v 2 = variance of intensity of vertical pixels $\mathrm{I}[\mathrm{r}-\mathrm{w}, \mathrm{c}] \ldots \mathrm{I}[\mathrm{r}+\mathrm{w}, \mathrm{c}]$;
v 3 = variance of intensity of diagonal pixels $I[r-w, c-w] \ldots[r+w, c+w]$;
$\mathrm{v} 4=$ variance of intensity of diagonal pixels $\mathrm{I}[\mathrm{r}-\mathrm{w}, \mathrm{c}+\mathrm{w}] . . . \mid[\mathrm{r}+\mathrm{w}, \mathrm{c}-\mathrm{w}]$;
return $\min (v 1, ~ v 2, ~ v 3, ~ v 4) ;$
\}

## Sparse Motion Estimation

- Search corresponding points
- Given an interesting point $P_{i}$ from $I_{t}$, take its neighbourhood in $I_{t}$ and find the best matching neighbourhood in $I_{t+\Delta t}$ under the assumption that the amount of movement is limited


This approach is also known as template matching

## Similarity Measures

- Cross-correlation (to be maximised)

$$
\mathrm{CC}(\Delta x, \Delta y)=\sum_{(x, y) \in T} I_{t}(x, y) \cdot I_{t+\Delta t}(x+\Delta x, y+\Delta y)
$$

- Sum of absolute differences (to be minimised)

$$
\operatorname{SAD}(\Delta x, \Delta y)=\sum_{(x, y) \in T}\left|I_{t}(x, y)-I_{t+\Delta t}(x+\Delta x, y+\Delta y)\right|
$$

- Sum of squared differences (to be minimised)

$$
\operatorname{SSD}(\Delta x, \Delta y)=\sum_{(x, y) \in T}\left[I_{t}(x, y)-I_{t+\Delta t}(x+\Delta x, y+\Delta y)\right]^{2}
$$

## Similarity Measures

- Mutual information (to be maximised)
$\operatorname{MI}(A, B)=\sum_{a} \sum_{b} P_{A B}(a, b) \log _{2}\left(\frac{P_{A B}(a, b)}{P_{A}(a) P_{B}(b)}\right)$
Subimages to compare:
$A \subset I_{t} \quad B \subset I_{t+\Delta t}$
Intensity probabilities:
$P_{A}(a) \quad P_{B}(b)$
Joint intensity probability:
$P_{A B}(a, b)$

$P_{B}(b)$



A
$P_{A}(a) \quad P_{A B}(a, b)$

## Dense Motion Estimation

## Dense Motion Estimation

- Assumptions:
- The object reflectivity and illumination do not change during the considered time interval
- The distance of the object from the camera and the light sources do not vary significantly over this interval
- Each small neighbourhood $N_{t}(x, y)$ at time $t$ is observed in some shifted position $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$ at time $t+\Delta t$
- These assumptions may not hold tight in reality, but provide useful computation and approximation


## Spatiotemporal Gradient

- Taylor series expansion of a function

$$
\begin{aligned}
& f(x+\Delta x)=f(x)+\frac{\partial f}{\partial x} \Delta x+\text { h.o.t } \Rightarrow \\
& f(x+\Delta x) \approx f(x)+\frac{\partial f}{\partial x} \Delta x
\end{aligned}
$$

- Multivariable Taylor series approximation

$$
\begin{equation*}
f(x+\Delta x, y+\Delta y, t+\Delta t) \approx f(x, y, t)+\frac{\partial f}{\partial x} \Delta x+\frac{\partial f}{\partial y} \Delta y+\frac{\partial f}{\partial t} \Delta t \tag{1}
\end{equation*}
$$

## Optical Flow Equation

Assuming neighbourhood $N_{t}(x, y)$ at time $t$ moves over vector $V=(\Delta x, \Delta y)$ to an identical neighbourhood $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$ at time $t+\Delta t$ leads to the optical flow equation:

$$
\begin{equation*}
f(x+\Delta x, y+\Delta y, t+\Delta t)=f(x, y, t) \tag{2}
\end{equation*}
$$



## Optical Flow Computation

Combining (1) and (2) yields the following constraint:

$$
\begin{aligned}
& \frac{\partial f}{\partial x} \Delta x+\frac{\partial f}{\partial y} \Delta y+\frac{\partial f}{\partial t} \Delta t=0 \Rightarrow \\
& \frac{\partial f}{\partial x} \frac{\Delta x}{\Delta t}+\frac{\partial f}{\partial y} \frac{\Delta y}{\Delta t}+\frac{\partial f}{\partial t} \frac{\Delta t}{\Delta t}=0 \Rightarrow \\
& \frac{\partial f}{\partial x} v_{x}+\frac{\partial f}{\partial y} v_{y}+\frac{\partial f}{\partial t}=0 \Rightarrow \\
& \nabla f \cdot v=-f_{t}
\end{aligned}
$$

where $v=\left(v_{x}, v_{y}\right)$ is the velocity or optical flow of $f(x, y, t)$ and $\nabla f=\left(f_{x}, f_{y}\right)=(\partial f / \partial x, \partial f / \partial y)$ is the gradient

## Optical Flow Computation

- The optical flow equation provides a constraint that can be applied at every pixel position
- However, the equation does not have unique solution and thus further constraints are required

For example, by using the optical flow equation for a group of adjacent pixels and assuming that all of them have the same velocity, the optical flow computation task amounts to solving a linear system of equations using the least-squares method

Many other solutions have been proposed (see references)

## Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

Assume the optical flow equation holds for all pixels $p_{i}$ in a certain neighbourhood and use the following notation:

$$
v=\left(v_{x}, v_{y}\right) \quad f_{x}=\frac{\partial f}{\partial x} \quad f_{y}=\frac{\partial f}{\partial y} \quad f_{t}=\frac{\partial f}{\partial t}
$$

Then we have the following set of equations:

$$
\begin{aligned}
& f_{x}\left(p_{1}\right) v_{x}+f_{y}\left(p_{1}\right) v_{y}=-f_{t}\left(p_{1}\right) \\
& f_{x}\left(p_{2}\right) v_{x}+f_{y}\left(p_{2}\right) v_{y}=-f_{t}\left(p_{2}\right)
\end{aligned}
$$

$$
f_{x}\left(p_{N}\right) v_{x}+f_{y}\left(p_{N}\right) v_{y}=-f_{t}\left(p_{N}\right)
$$

## Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

The set of equations can be rewritten as $A v=b$ where

$$
A=\left[\begin{array}{cc}
f_{x}\left(p_{1}\right) & f_{y}\left(p_{1}\right) \\
f_{x}\left(p_{2}\right) & f_{y}\left(p_{2}\right) \\
\vdots & \vdots \\
f_{x}\left(p_{N}\right) & f_{y}\left(p_{N}\right)
\end{array}\right] \quad v=\left[\begin{array}{l}
v_{x} \\
v_{y}
\end{array}\right] \quad b=\left[\begin{array}{c}
-f_{t}\left(p_{1}\right) \\
-f_{t}\left(p_{2}\right) \\
\vdots \\
-f_{t}\left(p_{N}\right)
\end{array}\right]
$$

This can be solved using the least-squares approach:

$$
A^{T} A v=A^{T} b \quad \Rightarrow \quad v=\left(A^{T} A\right)^{-1} A^{T} b
$$

## Optical Flow Example


https://www.youtube.com/watch?v=GIUDAZLfYhY

## References and Acknowledgements

- Chapter 8 of Szeliski 2010
- Chapter 9 of Shapiro and Stockman 2001
- Images drawn from the above references

