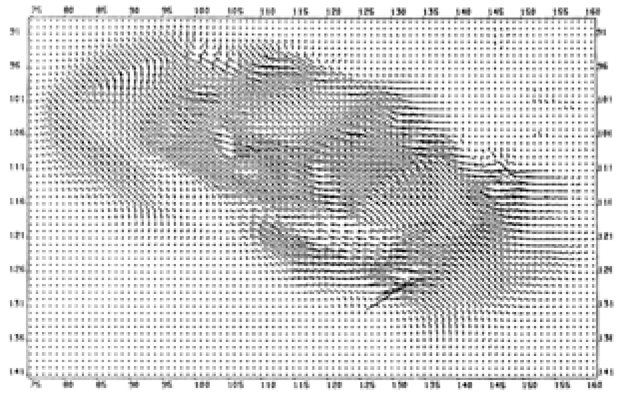
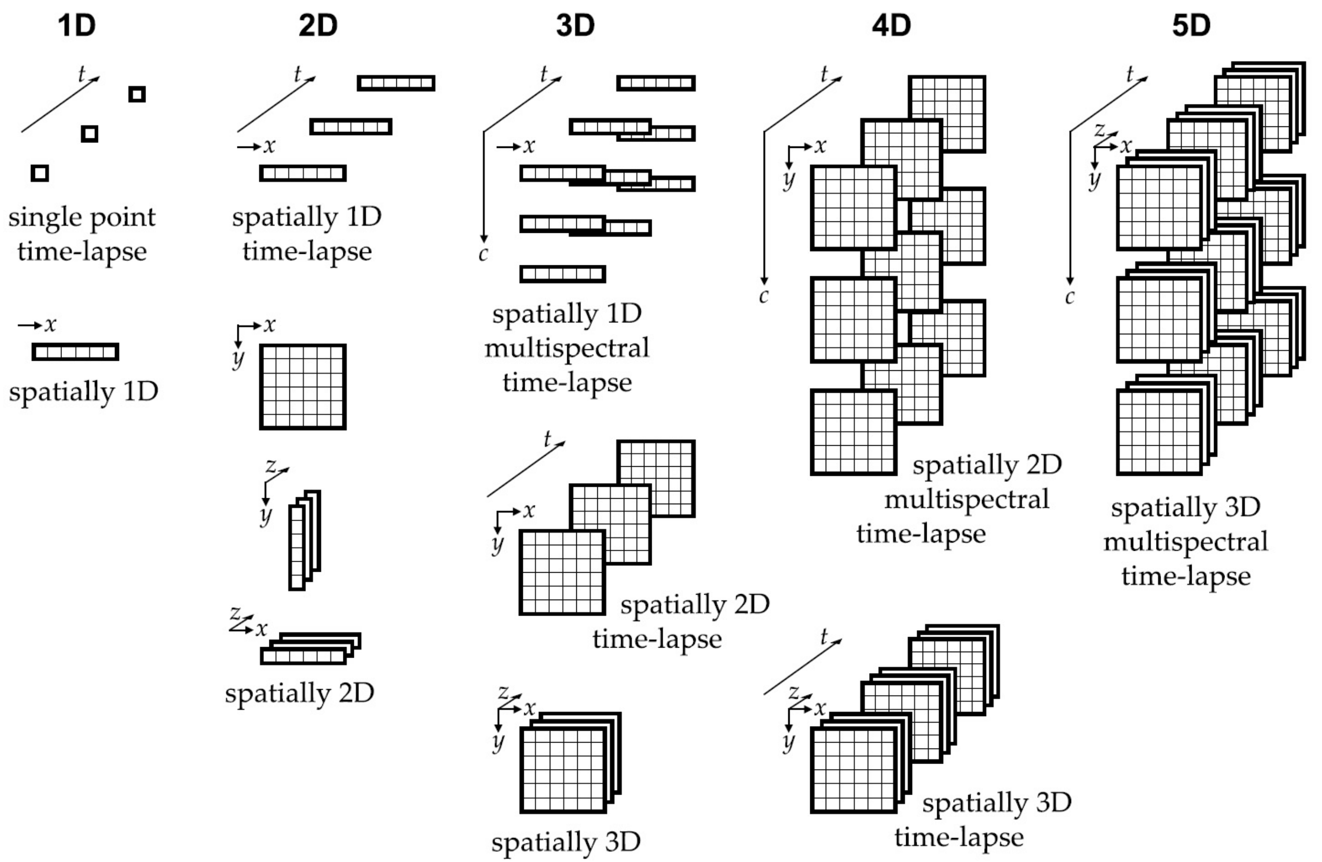


COMP9517: Computer Vision

Motion Estimation

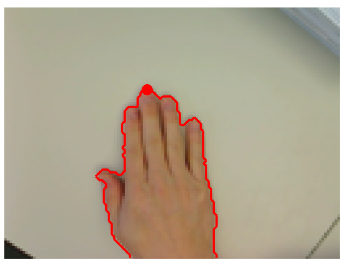
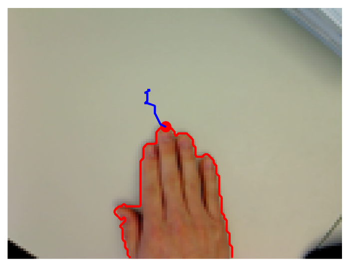
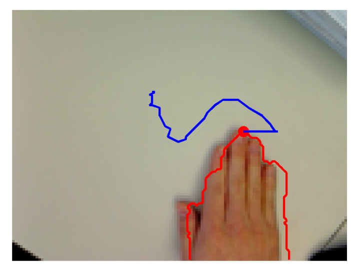
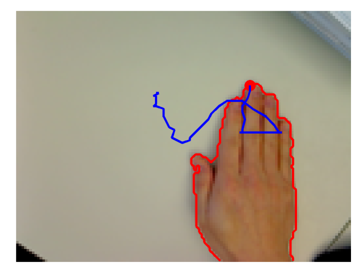
Introduction

- Adding the time dimension to the image formation



Introduction

- A changing scene may be observed and analysed via a sequence of images

Introduction

- Changes in an image sequence provide features for
  - Detecting objects that are moving
  - Computing trajectories of moving objects
  - Performing motion analysis of moving objects
  - Recognising objects based on their behaviours
  - Computing the motion of the viewer in the world
  - Detecting and recognising activities in a scene

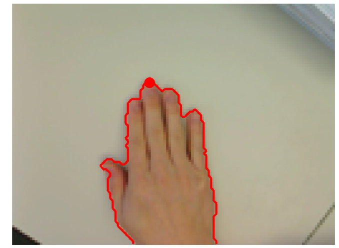
Applications

- **Motion-based recognition**
  – Human identification based on gait, automatic object detection
- **Automated surveillance**
  – Monitoring a scene to detect suspicious activities or unlikely events
- **Video indexing**
  – Automatic annotation and retrieval of videos in multimedia databases
- **Human-computer interaction**
  – Gesture recognition, eye gaze tracking for data input to computers
- **Traffic monitoring**
  – Real-time gathering of traffic statistics to direct traffic flow
- **Vehicle navigation**
  – Video-based path planning and obstacle avoidance capabilities

Scenarios

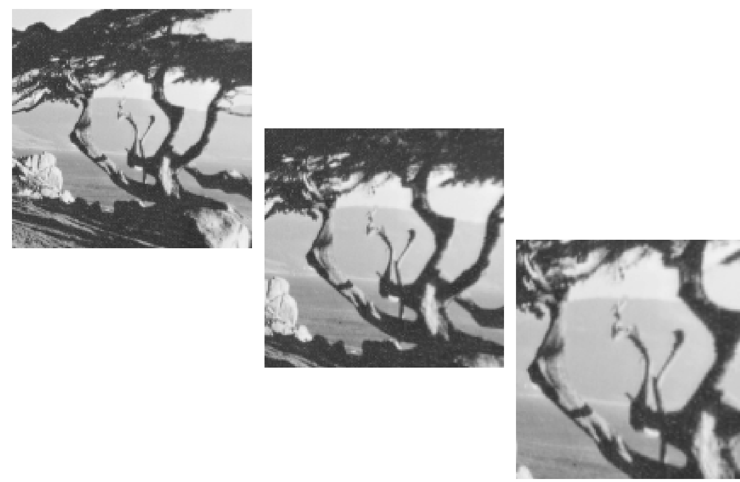
- **Still camera**

  Constant background with
  - Single moving object
  - Multiple moving objects



- **Moving camera**

  Relatively constant scene with
  - Coherent scene motion
  - Single moving object
  - Multiple moving objects



Topics

- **Change detection**

  Using *image subtraction* to detect changes in scenes

- **Sparse motion estimation**

  Using *template matching* to estimate local displacements

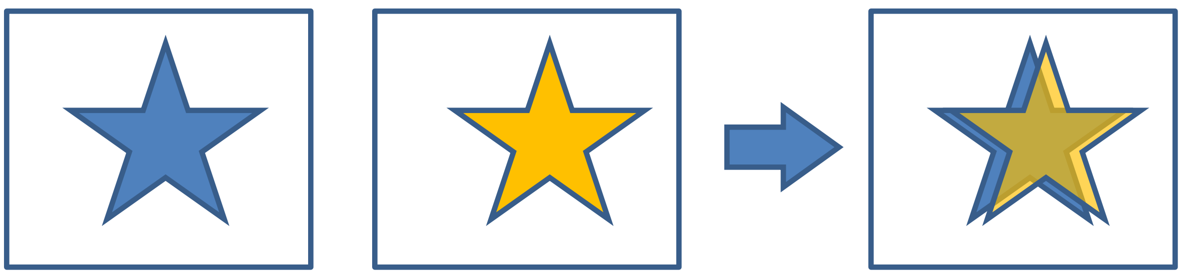
- **Dense motion estimation**

  Using *optical flow* to compute a dense motion vector field

Change Detection

Change Detection

- Detecting an object moving across a constant background
- The forward and rear edges of the object advance only a few pixels per frame



- By subtracting the image I_t from the previous image I_{t-1} the edges should be evident as the only pixels significantly different from zero

Image Subtraction

Step: Derive a background image from a set of video frames at the beginning of the video sequence

Performance **E**valuation of **T**racking and **S**urveillance (PETS) 2009 Benchmark

Image Subtraction

Step: Subtract the background image from each subsequent frame to create a difference image

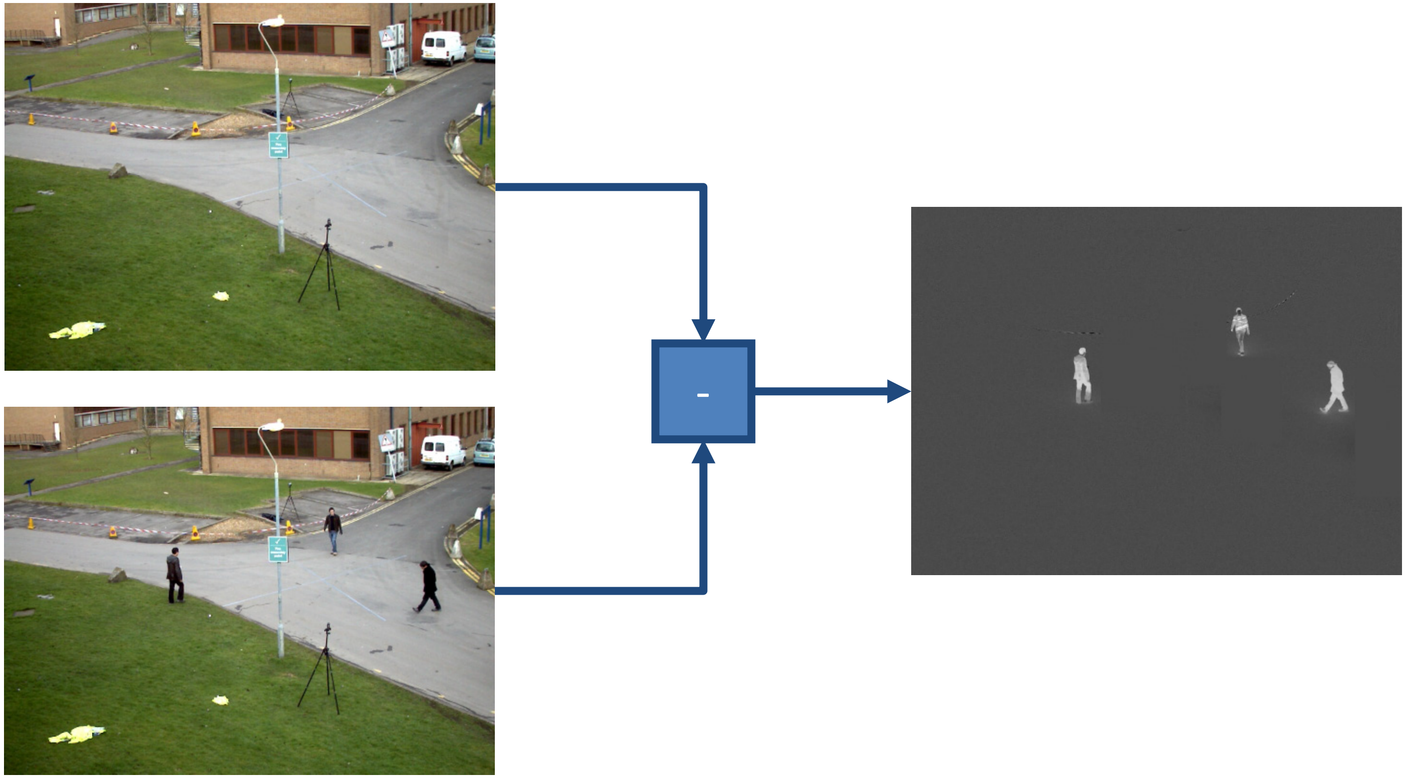


Image Subtraction

Step: Threshold and enhance the difference image to fuse neighbouring regions and remove noise

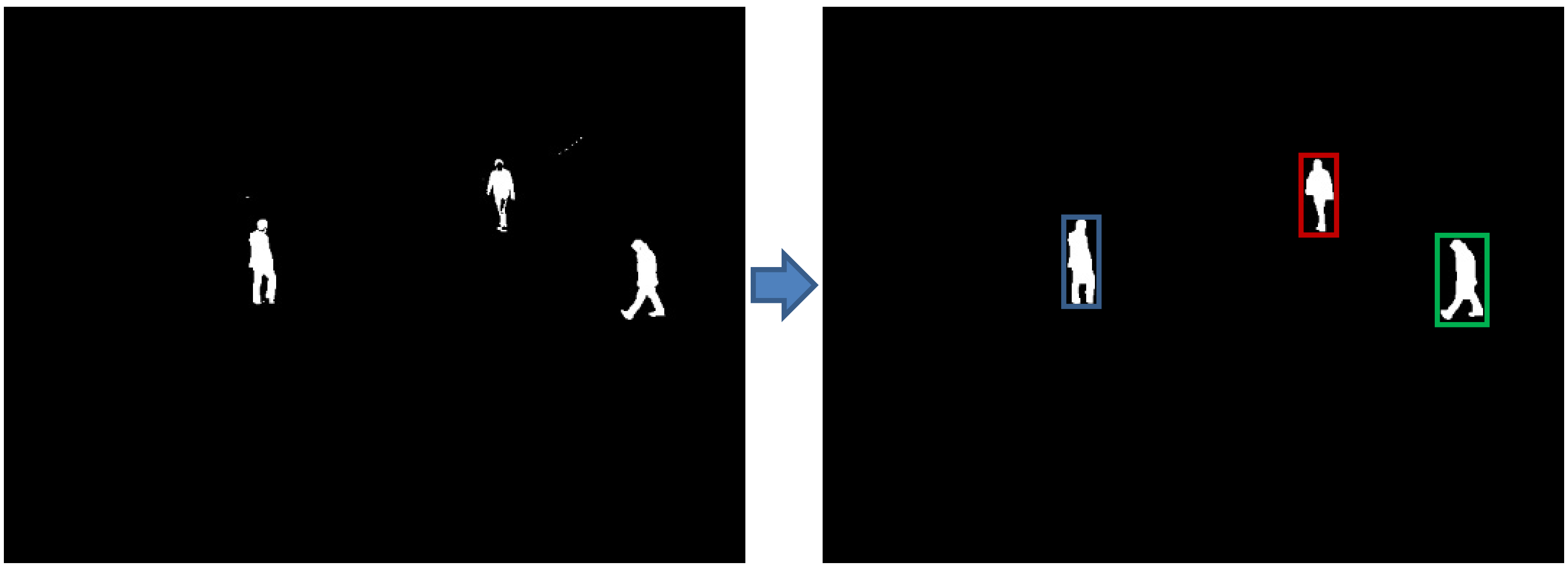
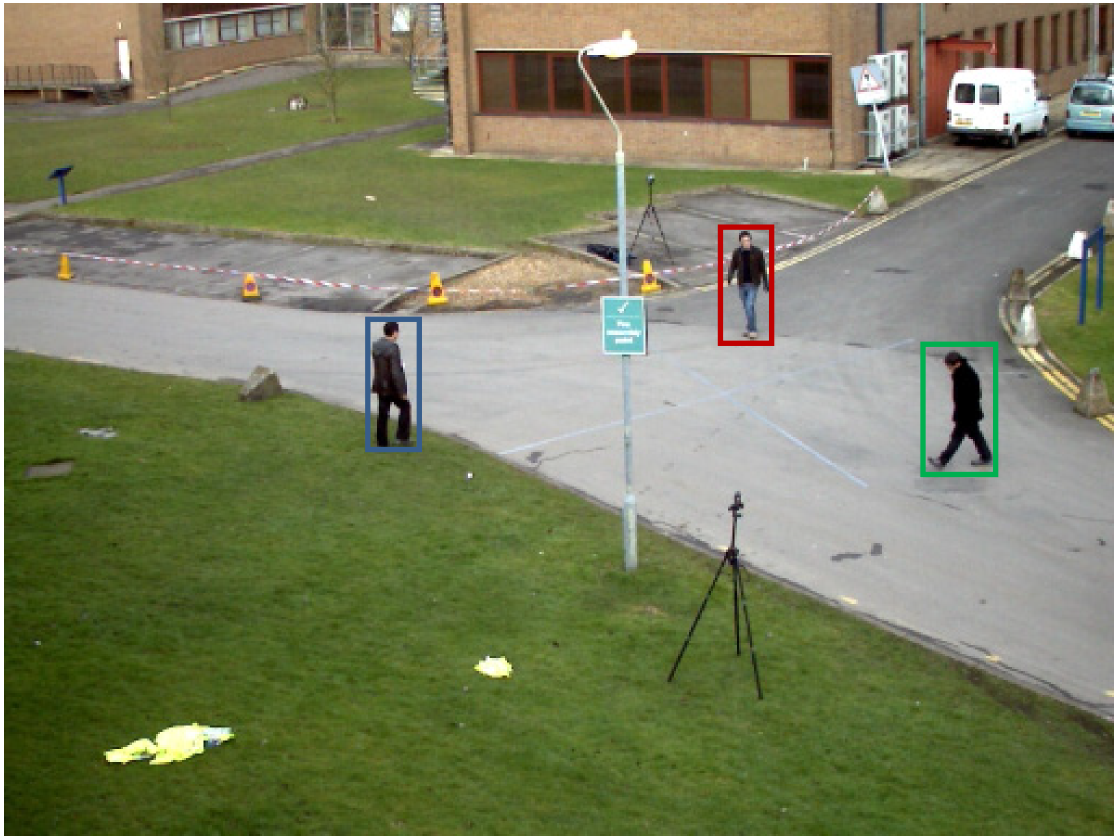


Image Subtraction

Detected bounding boxes overlaid on input frame



Change Detection

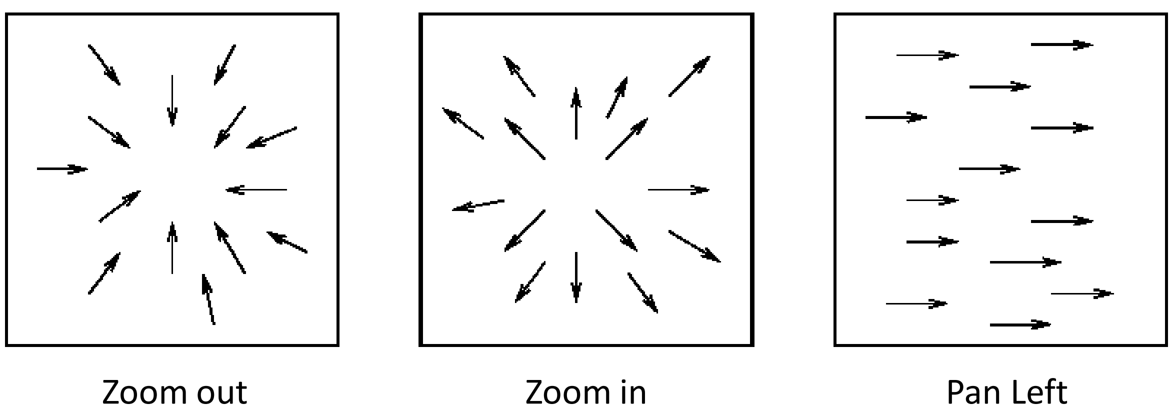
- Image subtraction algorithm
  - Input: images I_t and $I_{t-\Delta t}$ (or a model image)
  - Input: an intensity threshold τ
  - Output: a binary image I_{out}
  - Output: a set of bounding boxes B

1. For all pixels [r, c] in the input images,
   set $I_{out}[r, c] = 1$ if $(|I_t[r, c] - I_{t-\Delta t}[r, c]| > \tau)$
   set $I_{out}[r, c] = 0$ otherwise
2. Perform connected components extraction on I_{out}
3. Remove small regions in I_{out} assuming they are noise
4. Perform a closing of I_{out} using a small disk to fuse neighbouring regions
5. Compute the bounding boxes of all remaining regions of changed pixels
6. Return $I_{out}[r, c]$ and the bounding boxes B of regions of changed pixels

Sparse Motion Estimation

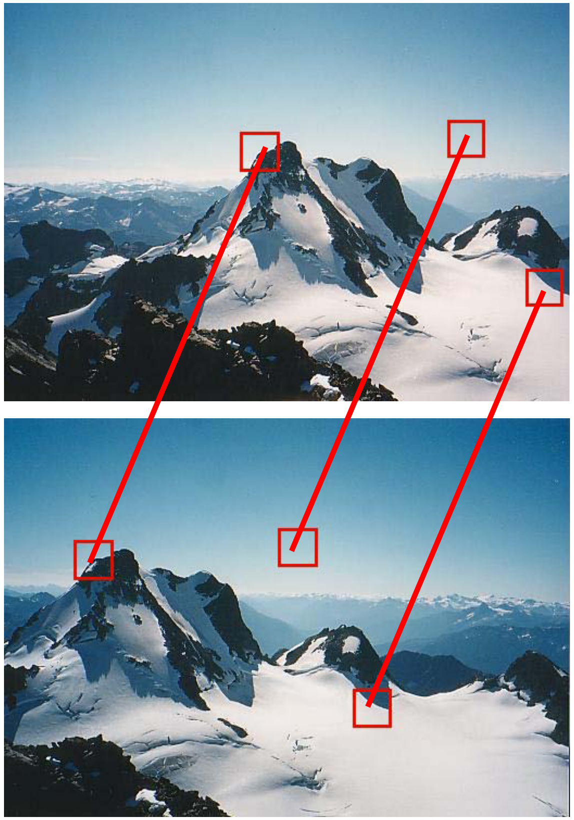
Motion Vector

- A motion field is a 2D array of 2D vectors representing the motion of 3D scene points
- A motion vector in the image represents the displacement of the image of a moving 3D point
  - Tail at time t and head at time t+Δt
  - Instantaneous velocity estimate at time t



Sparse Motion Estimation

- A sparse motion field can be computed by identifying pairs of points that correspond in two images taken at times t and t+Δt
- Assumption: intensities of interesting points and their neighbours remain nearly constant over time
- Two steps:
  - Detect interesting points at t
  - Search corresponding points at t+Δt



Sparse Motion Estimation

- Detect interesting points
  - Image filters
    - Canny edge detector
    - Hessian ridge detector
    - Harris corner detector
    - Scale invariant feature transform (SIFT)
    - Fully convolutional neural network (FCN)
  - Interest operator
    - Computes intensity variance in the vertical, horizontal and diagonal directions
    - Interest point if the minimum of these four variances exceeds a threshold

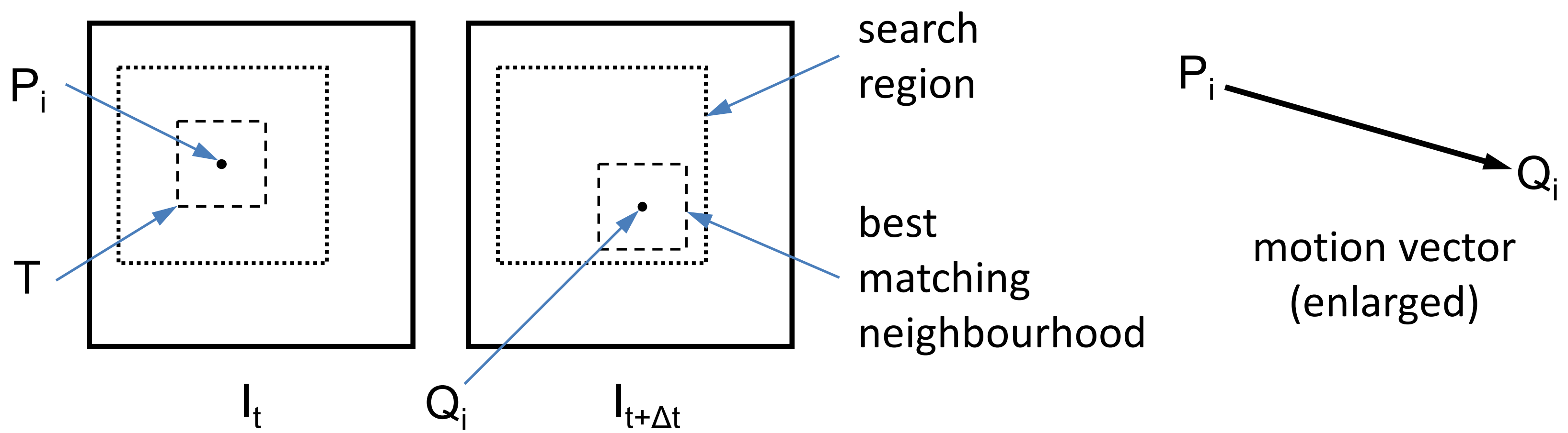
Detect Interesting Points

```
Procedure detect_interesting_points(I,V,w,t) {
    for (r = 0 to MaxRow – 1)
        for (c = 0 to MaxCol – 1)
            if (I[r,c] is a border pixel) break;
            else if (interest_operator(I,r,c,w) >= t)
                add (r,c) to set V;
}

Procedure interest_operator (I,r,c,w) {
    v1 = variance of intensity of horizontal pixels I[r,c-w]…I[r,c+w];
    v2 = variance of intensity of vertical pixels I[r-w,c]…I[r+w,c];
    v3 = variance of intensity of diagonal pixels I[r-w,c-w]…I[r+w,c+w];
    v4 = variance of intensity of diagonal pixels I[r-w,c+w]…I[r+w,c-w];
    return min(v1, v2, v3, v4);
}
```

Sparse Motion Estimation

- Search corresponding points
  - Given an interesting point P_i from I_t, take its neighbourhood in I_t and find the best matching neighbourhood in $I_{t+\Delta t}$ under the assumption that the amount of movement is limited



This approach is also known as template matching

Similarity Measures

- Cross-correlation (to be maximised)

$$\mathrm{CC}(\Delta x, \Delta y) = \sum_{(x,y)\in T} I_t\left(x, y\right) \cdot I_{t+\Delta t}\left(x + \Delta x, y + \Delta y\right)$$

- Sum of absolute differences (to be minimised)

$$\mathrm{SAD}(\Delta x, \Delta y) = \sum_{(x,y)\in T} \left| I_t\left(x, y\right) - I_{t+\Delta t}\left(x + \Delta x, y + \Delta y\right) \right|$$

- Sum of squared differences (to be minimised)

$$\mathrm{SSD}(\Delta x, \Delta y) = \sum_{(x,y)\in T} \left[I_t\left(x, y\right) - I_{t+\Delta t}\left(x + \Delta x, y + \Delta y\right) \right]^2$$

Similarity Measures

- Mutual information (to be maximised)

$$\mathrm{MI}(A,B)=\sum_a\sum_b P_{AB}(a,b)\log_2\left(\frac{P_{AB}(a,b)}{P_A(a)P_B(b)}\right)$$

Subimages to compare:

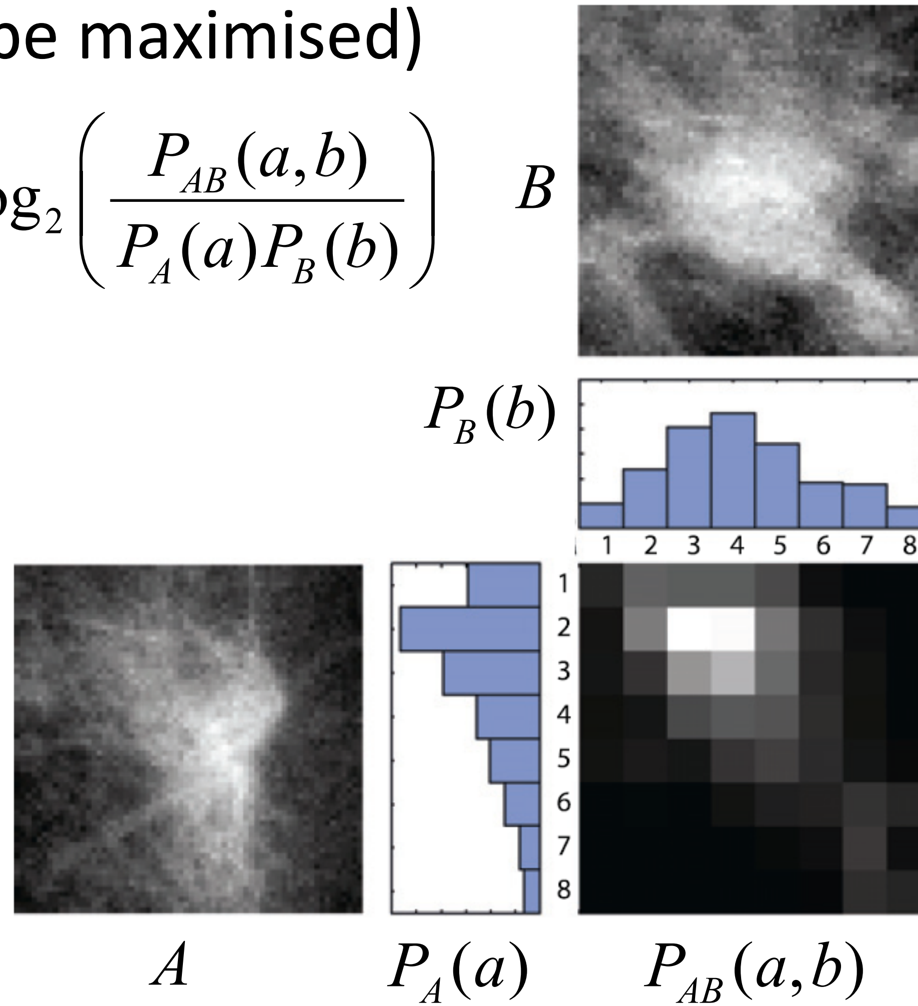
$$A\subset I_t \qquad B\subset I_{t+\Delta t}$$

Intensity probabilities:

$$P_A(a) \qquad P_B(b)$$

Joint intensity probability:

$$P_{AB}(a,b)$$



Dense Motion Estimation

Dense Motion Estimation

- Assumptions:
  - The object reflectivity and illumination do not change during the considered time interval
  - The distance of the object to the camera and the light sources does not vary significantly over this interval
  - Each small neighbourhood $N_t(x,y)$ at time t is observed in some shifted position $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$ at time $t+\Delta t$
- These assumptions may not hold tight in reality, but provide useful computation and approximation

Spatiotemporal Gradient

- Taylor series expansion of a function

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \text{h.o.t} \quad \Rightarrow$$

$$f(x+\Delta x) \approx f(x) + \frac{\partial f}{\partial x}\Delta x$$

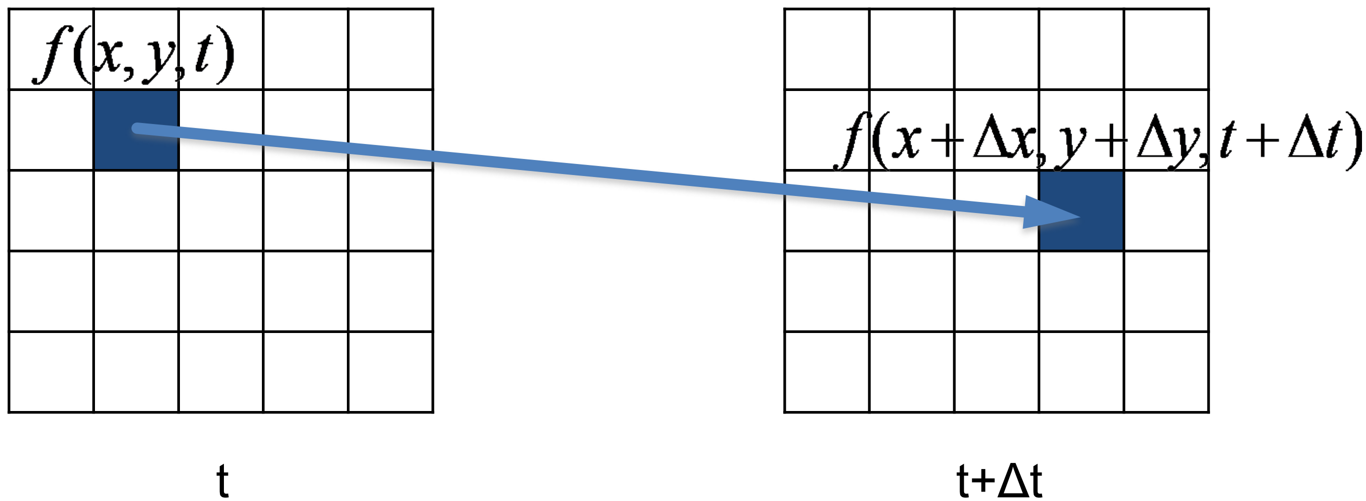
- Multivariable Taylor series approximation

$$f(x+\Delta x, y+\Delta y, t+\Delta t) \approx f(x,y,t) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial t}\Delta t \qquad (1)$$

Optical Flow Equation

Assuming neighbourhood $N_t(x, y)$ at time t moves over vector $V=(\Delta x, \Delta y)$ to an identical neighbourhood $N_{t+\Delta t}(x+\Delta x, y+\Delta y)$ at time $t+\Delta t$ leads to the optical flow equation:

$$f(x+\Delta x, y+\Delta y, t+\Delta t) = f(x, y, t) \qquad (2)$$



Optical Flow Computation

Combining (1) and (2) yields the following constraint:

$$\frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial t}\Delta t = 0 \Rightarrow$$

$$\frac{\partial f}{\partial x}\frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y}\frac{\Delta y}{\Delta t} + \frac{\partial f}{\partial t}\frac{\Delta t}{\Delta t} = 0 \Rightarrow$$

$$\frac{\partial f}{\partial x}v_x + \frac{\partial f}{\partial y}v_y + \frac{\partial f}{\partial t} = 0 \Rightarrow$$

$$\nabla f \cdot v = -f_t$$

where $v = (v_x, v_y)$ is the velocity or *optical flow* of $f(x, y, t)$
and $\nabla f = (f_x, f_y) = (\partial f / \partial x, \partial f / \partial y)$ is the gradient

Optical Flow Computation

- The optical flow equation provides a constraint that can be applied at every pixel position

- However, the equation does not have unique solution and thus further constraints are required

  For example, by using the optical flow equation for a group of adjacent pixels and assuming that all of them have the same velocity, the optical flow computation task amounts to solving a linear system of equations using the least-squares method

  Many other solutions have been proposed (see references)

Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

Assume the optical flow equation holds for all pixels p_i in a certain neighbourhood and use the following notation:

$$v = (v_x, v_y) \qquad f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y} \qquad f_t = \frac{\partial f}{\partial t}$$

Then we have the following set of equations:

$$\begin{aligned} f_x(p_1)v_x + f_y(p_1)v_y &= -f_t(p_1) \\ f_x(p_2)v_x + f_y(p_2)v_y &= -f_t(p_2) \\ \vdots \qquad\qquad \vdots \qquad\qquad & \qquad \vdots \\ f_x(p_N)v_x + f_y(p_N)v_y &= -f_t(p_N) \end{aligned}$$

Optical Flow Computation

- Example: Lucas-Kanade approach to optical flow

The set of equations can be rewritten as $Av = b$ where

$$A = \begin{bmatrix} f_x(p_1) & f_y(p_1) \\ f_x(p_2) & f_y(p_2) \\ \vdots & \vdots \\ f_x(p_N) & f_y(p_N) \end{bmatrix} \qquad v = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \qquad b = \begin{bmatrix} -f_t(p_1) \\ -f_t(p_2) \\ \vdots \\ -f_t(p_N) \end{bmatrix}$$

This can be solved using the least-squares approach:

$$A^T A v = A^T b \qquad \Rightarrow \qquad v = \left(A^T A \right)^{-1} A^T b$$

Optical Flow Example

https://www.youtube.com/watch?v=GIUDAZLfYhY

References and Acknowledgements

- Chapter 8 of Szeliski 2010
- Chapter 9 of Shapiro and Stockman 2001
- Some images drawn from the above references

Example exam question

Which one of the following statements about motion analysis is incorrect?

A. Detection of moving objects by subtraction of successive images in a video works best if the background is constant.

B. Sparse motion estimation in a video can be done by template matching and minimising the mutual information measure.

C. Dense motion estimation using optical flow assumes that each small neighbourhood remains constant over time.

D. Optical flow provides an equation for each pixel but requires further constraints to solve the equation uniquely.