GSOE9210 Engineering Decisions

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Game theory: introduction

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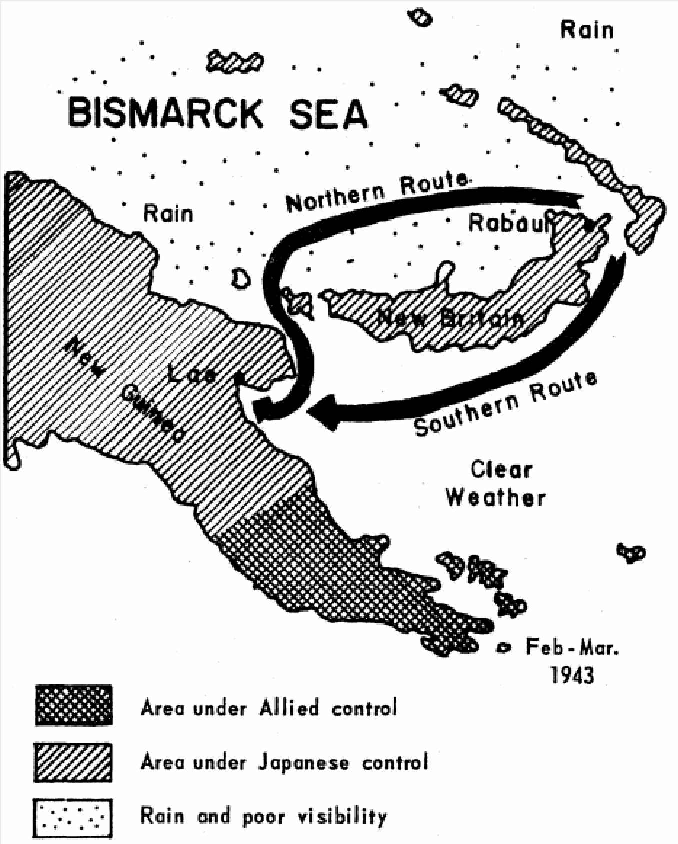
Military strategy



Example (Battle of Bismarck)

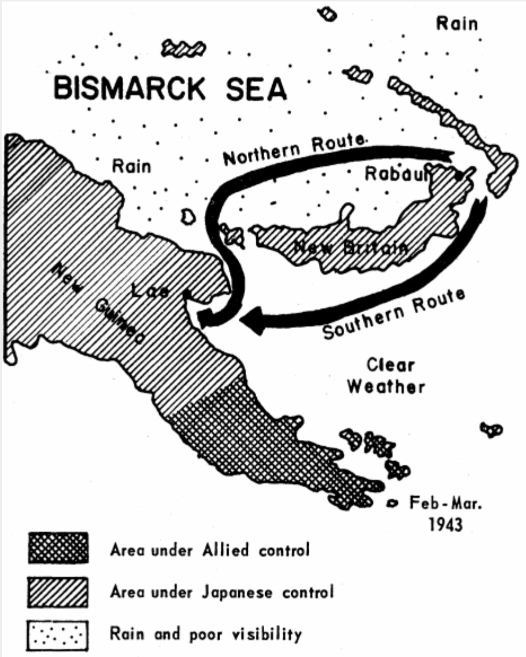
Battle theatre:

- Two possible routes for *Convoy* from Rabaul to Lae, each taking three days to complete
- *Allies*' search aircraft can concentrate on either route
- Bad weather on Northern route makes search difficult
- Once *Convoy* spotted, bombers deployed to attack it

Decisions:

- *Convoy*: which route?
- *Allies*: search where?

Game elements



This setting involves:

- more than one agent (called *players*): Allies and Convoy
- *moves/strategies* for each player: choice of route for convoy; search area for Allies
- *outcomes* that *co-depend* on the strategies of both players (*play*): the four possible scenarios above
- *preferences* over outcomes represented by *payoffs* for each player: number of days convoy is bombed

Game theory

Definition (Game)

A *game* is any setting in which there are more than one decision-makers, called *players*, and in which the outcomes may co-depend on the actions/strategies of all players.

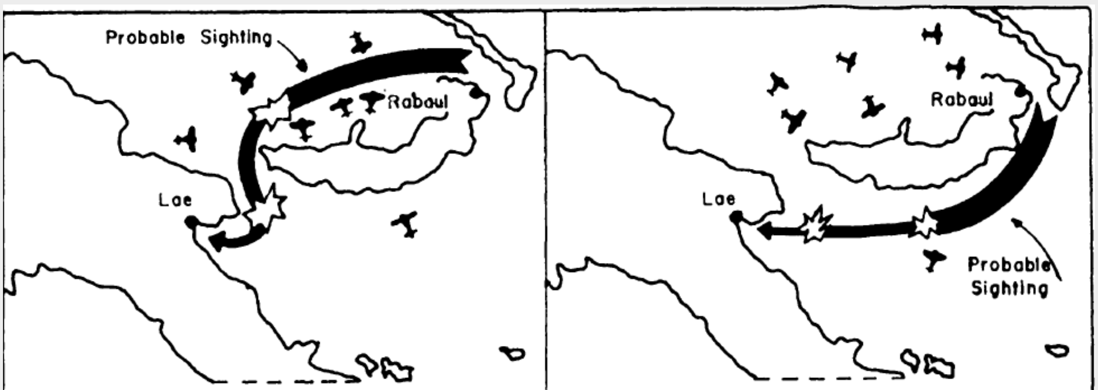
- A *solution* of the game is a any combination of strategies/outcomes which result from *rational play* by all players

Aim of Game Theory

The aim of game theory is to identify solutions to games.

Strategic analysis

Allies' option 1: Concentrate search in the North.



Kenny Strategy: Concentrate reconnaissance on northern route.

Japanese Strategy: Sail northern route.

Estimated Outcome: Although reconnaissance would be hampered by poor visibility, the convoy should be discovered by the second day, which would permit two days of bombing.

TWO DAYS OF BOMBING

Kenney Strategy: Concentrate reconnaissance on northern route.

Japanese Strategy: Sail southern route.

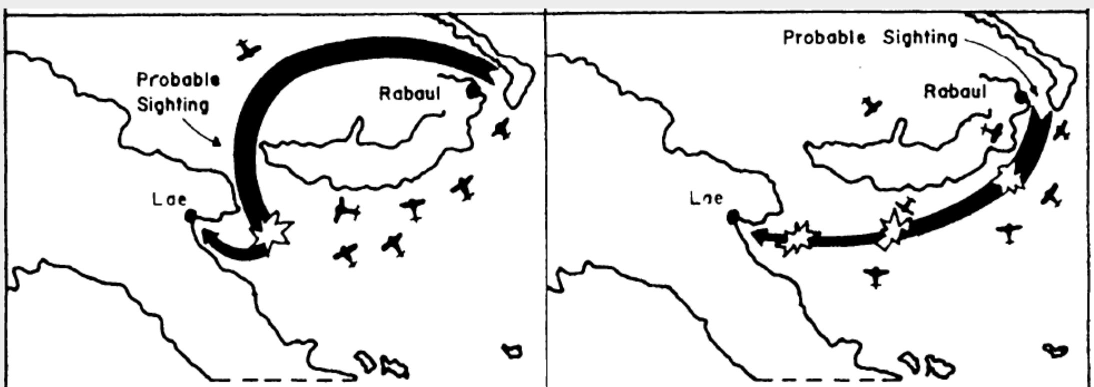
Estimated Outcome: The convoy would be sailing in clear weather. However, with limited reconnaissance aircraft in this area, the convoy might be missed on the first day. Convoy should be sighted by second day, to permit two days of bombing.

TWO DAYS OF BOMBING

Game theory Representing games

Strategic analysis

Allies' option 2: Concentrate search to the South.



Kenney Strategy: Concentrate reconnaissance on southern route.

Japanese Strategy: Sail northern route.

Estimated Outcome: With poor visibility and limited reconnaissance, Kenney could not expect the convoy to be discovered until it broke out into clear weather on third day. This would permit only one day of bombing.

ONE DAY OF BOMBING

Kenney Strategy: Concentrate reconnaissance on southern route.

Japanese Strategy: Sail southern route.

Estimated Outcome: With good visibility and concentrated reconnaissance in the area, the convoy should be sighed almost as soon as it sailed from Rabaul. This would allow three days of bombing.

THREE DAYS OF BOMBING

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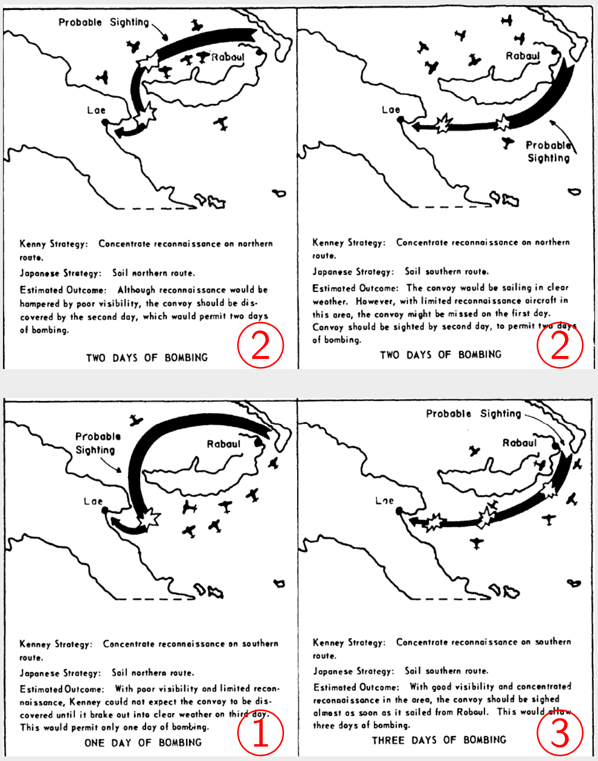
Battle: table representation

n s

N

S



Convoy

Allies		n	s
	N	2	2
	S	1	3

- Rows = Allies' general's (A) actions/strategies
  Columns = Convoy's general's (C) actions
- Abstraction: row player and column player
- Payoffs: number of bombing days

Game theory FAQ

Question

Haven't we seen this already?

What's new/different?

- One source of uncertainty due to others' strategies . . .
- Information about others' preferences
- Additional information: assumption that other agents rational!

A simple game: pursuit and evasion

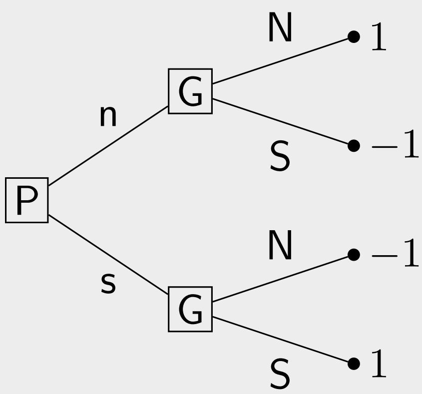
Example (Pursue and evade)

A prisoner (P) is planning an escape from prison. There are two possible escape routes: in the prison's North or South wings. A prison guard (G) is on watch. The guard can patrol one wing but not both.

Prison escape: game tree analysis

- Represent this scenario as a *game tree*:

  From G's viewpoint:

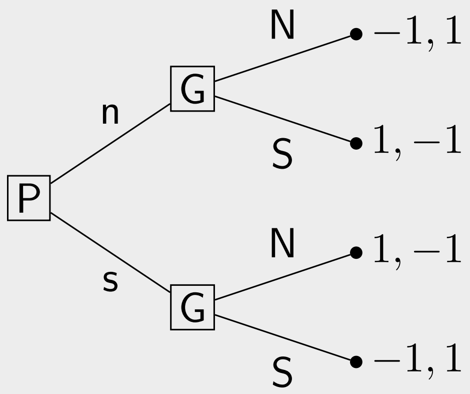


  where in outcome e the prisoner escapes, and in c he's caught.
- Each player has different *payoff, or utility, functions* for the outcomes; for each player p (here $p \in \{\mathsf{P}, \mathsf{G}\}$):

$$u_p : \Omega \to \mathbb{R}$$

Prison escape: game tree

- Combine each player's payoffs:



- Each outcome has a *payoff vector*; one value for each player:

$$(u_{\mathsf{P}}(\omega), u_{\mathsf{G}}(\omega))$$

- In this case payoffs are complementary: *i.e.*, $u_{\mathsf{P}}(\omega) + u_{\mathsf{G}}(\omega) = 0$. Such games are called *zero-sum games*

Games in extensive form

Definition (Game tree)

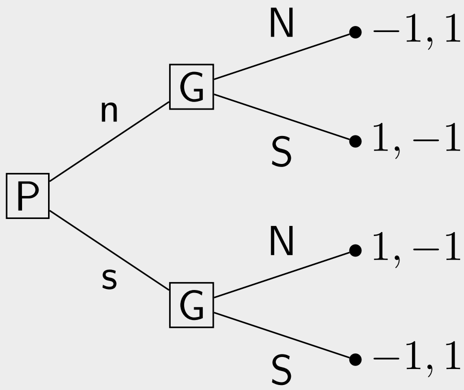
A *game tree* is also called the *extensive form* of a game.

Game trees allow fine modelling of games:

- *individual moves* at different stages for each player
- *turn structure*: players make moves at different stages: *e.g.*, alternating, simultaneous, *etc.*
- *information states*, or *epistemic states* ('states of knowledge'), of players at each decision point
- *contingent/conditional actions/strategies* for each player which depend on its epistemic state: *e.g.*, if prisoner moves North, I'll move North too

Prison escape: epistemic state

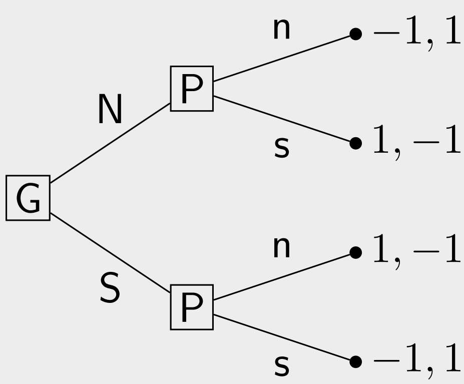
Case 1: Guard observes prisoner's movements:



- Additional knowledge/information about the prisoner's move gives guard an advantage
- Guard's optimal strategy: "follow prisoner's move"; *i.e.*, if P moves n, then move N; if P moves s, then move S

Escape reversed

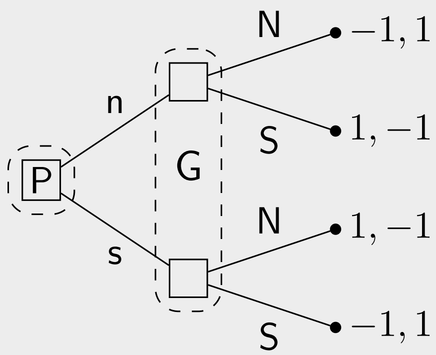
Case 2: Prisoner observes guard's movements:



- Additional knowledge gives advantage to the prisoner
- Optimal strategy: move opposite the guard; *i.e.*, if G moves n, then move s; if G moves s, then move n.

Modelling information

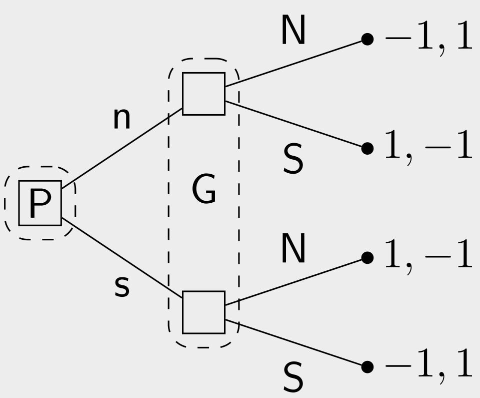
Case 3: Neither observes the other's move (*e.g.*, simultaneous moves):

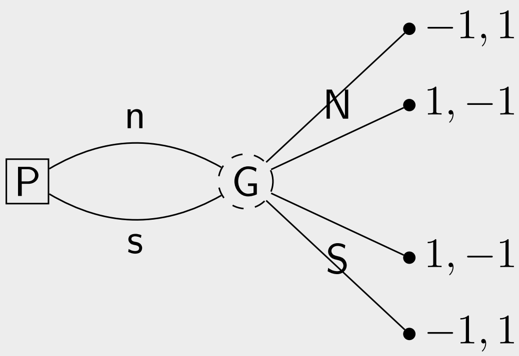


Definition (Information set)

An *information set* is a set of decision nodes that are epistemically indistinguishable by an agent. An information set defines an agent's epistemic state at some decision point. In a game of *perfect information* every information set has only a single node; *i.e.*, is a singleton set.

Epistemic modelling



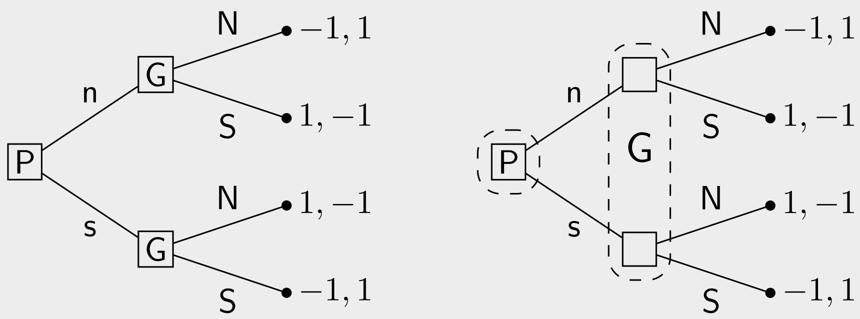


- The *game graph* on the right is an alternative representation of prisoner escape game in Case 3
- Here P's action is unknown to G: *i.e.*, both possibilities lead to same epistemic state for G
- G's moves are *non-deterministic* in sense that same action leads to different outcomes

Normal form

Definition

A game matrix is called the *normal (strategic) form* of a game.



What do the normal forms of the game trees above look like?

		G: N	G: S
P	n	$-1, 1$	$1, -1$
	s	$1, -1$	$-1, 1$

Modelling information

		G: N	G: S
P	n	$-1, 1$	$1, -1$
	s	$1, -1$	$-1, 1$

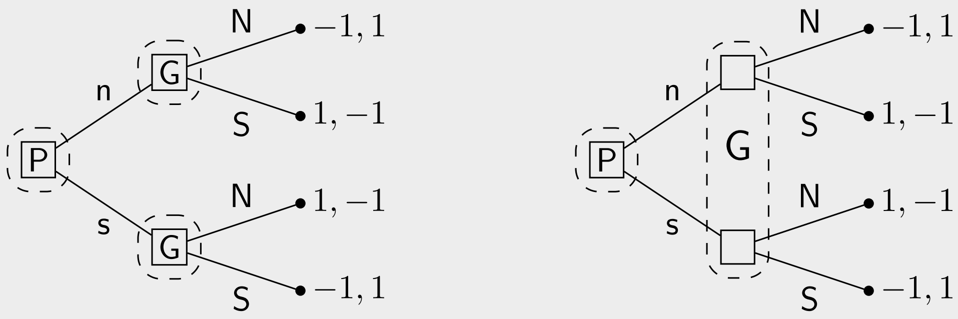
- By observing P's move in Case 1, G should have a 'winning strategy'; *i.e.*, one that always yields payoff 1 to G
- Let F be guard's optimal strategy: "follow prisoner's move"

		G: N	G: S	G: F
P	n	$-1, 1$	$1, -1$	$-1, 1$
	s	$1, -1$	$-1, 1$	$-1, 1$

Possible strategies

Definition

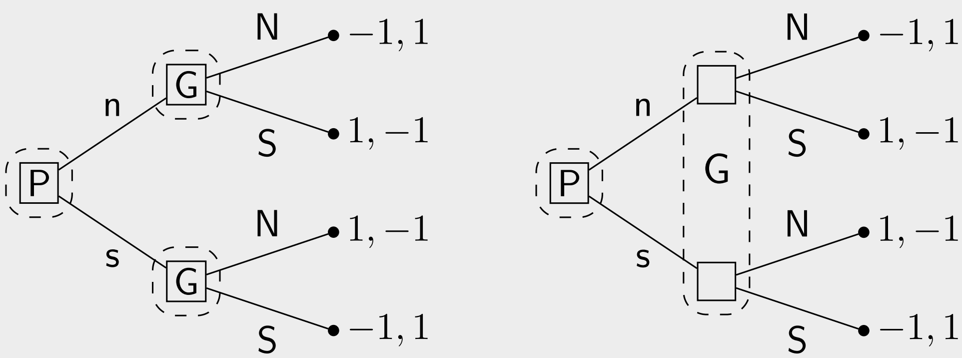
A *strategy* for an agent is the specification of a unique move in each of its (reachable) information sets (epistemic states).



Possible strategies for G in Case 1:

- if n, then N; if s, then N
- if n, then N; if s, then S
- if n, then S; if s, then N
- if n, then S; if s, then S

Normal form

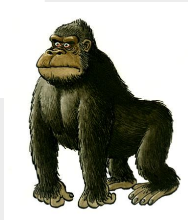


		G: n/N s/N	n/N s/S	n/S s/N	n/S s/S
P	n	$-1, 1$	$-1, 1$	$1, -1$	$1, -1$
	s	$1, -1$	$-1, 1$	$1, -1$	$-1, 1$

		G: N	S
P	n	$-1, 1$	$1, -1$
	s	$1, -1$	$-1, 1$

Meet Alice and Bob

Bob



Alice

Example: Alice and Bob

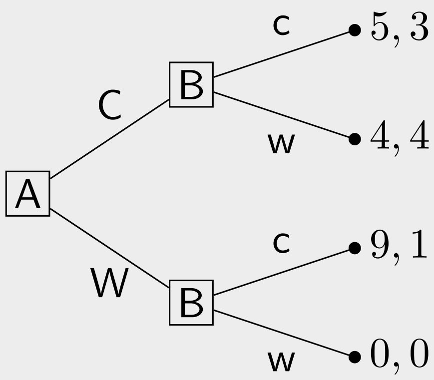
Example (Alice, Bob, and a coconut)

Alice (A) and Bob (B) are at a coconut tree which has only one coconut worth 10 kilocalories (kc) of energy in total. To get the coconut, one (or both) must climb the tree to shake it loose. It would take Alice some effort (2kc) to climb the tree, whereas Bob's effort is negligible.

If Bob climbs (c) the tree and Alice waits (W) below then Alice will get to the coconut first, eating most of it (9kc worth) and leaving only a small portion for Bob. If Alice climbs (C) and Bob waits (w) below then Bob will get to it first and eat his fill (4kc worth) before Alice gets down and takes it off him. If both climb up, Bob will climb down quicker and eat some (3kc worth) before Alice gets down and takes the rest.

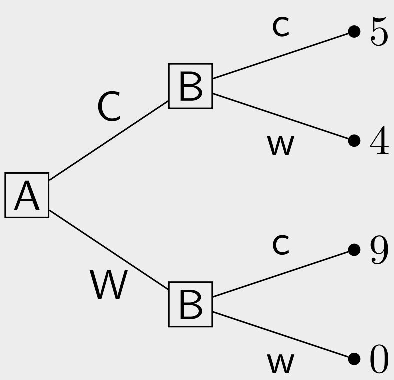
Game structure: Alice moves first

- Suppose Alice moves first; in which case Bob will gain information about Alice's move.



- What should Alice do?
  - Wait below hoping for 9kc and risk 0kc?
  - Climb herself, settling for something in between?

Games vs single-agent decisions

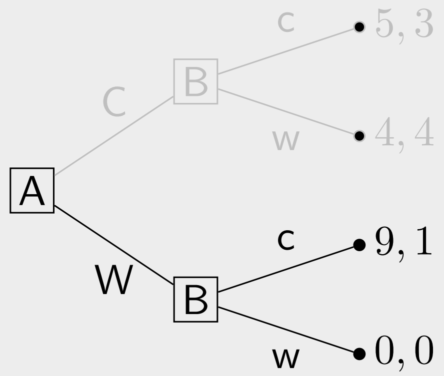


	c	w
C	5	4
W	9	0

- From Alice's perspective the 'decision table' would look like the one above
- Alice might use one of the decision rules under ignorance as she doesn't know what Bob will do; *e.g.*, *Maximin* (C)
- But Alice isn't ignorant about Bob! Alice knows Bob is rational (*i.e.*, will try to maximise utility)

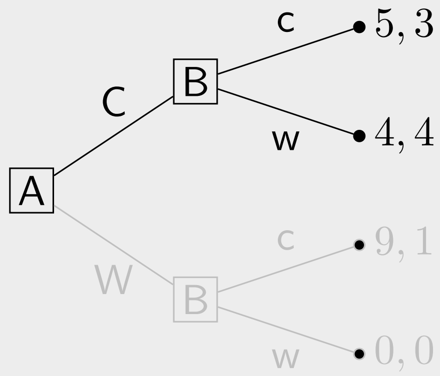
Alice's 'What if . . . ' analysis

'if I wait . . . '



'. . . Bob will climb'

'if I climb . . . '



'. . . Bob will wait'

Alice's conclusion

Alice's best strategy, considering Bob's rational response, should be to Wait in preference to Climbing (payoff to Alice of 9 compared to 4).

Strategies and counter-strategies

- If Alice moves first, Bob has more information, and hence more strategic options; *i.e.*, Bob's *possible pure strategies* are:
  - Regardless of whether Alice climbs or waits, I will wait
  - Regardless of whether Alice climbs or waits, I will climb
  - I will do the same as Alice: *i.e.*, if Alice climbs, I will climb; if Alice waits I will wait
  - I will do the opposite of Alice: *i.e.*, if Alice climbs, I will wait; if Alice waits I will climb
- If Alice waits, then Bob's best counter-strategy is to climb
- If Alice climbs, then Bob's best counter-strategy is to wait
- Combining these, Bob's optimal strategy is to do the opposite of what Alice does

Additional information of games

- Game matrix:

		B			
		W/w C/w	W/w C/c	W/c C/w	W/c C/c
A	C	$4, 4$	$5, 3$	$4, 4$	$5, 3$
	W	$0, 0$	$0, 0$	$9, 1$	$9, 1$

- Bob's dominant strategy is: "do the opposite of what Alice does"; *i.e.*, "if Alice waits, then I climb; if Alice climbs, I wait": W/c C/w

Reasoning about other agents' preferences

- Previous example shows why multi-agent decisions are more complex than single agent decisions
- Epistemic states affect available strategies
- Multi-agent decisions should incorporate the preferences and epistemic state of the other agents; *e.g.*, Alice's "what if . . ." analysis of Bob's response to her move

Conclusion

Reasoning about other players' preferences might improve the outcome for each player.

Game solutions

Definition (Plays and solutions)

In two-player games, a *play* is a pair (s_1, s_2) consisting of a strategy for each player. A play uniquely determines an *outcome* to the game. For n-player games this generalises to n-tuples $(s_1, s_2, \ldots, s_n)$. The outcome of 'rational' strategies from each player is called a *solution* to the game.

- Game theory is about developing methods and techniques to identify solutions to games
- Dominance can help simplify the problem based on the agents' preferences
- Do all games have solutions? (Existence)
- Can a game have more than one solution? (Uniqueness)

Two-player strictly competitive games

Definition (Two-player strictly competitive game)

A *two-player strictly competitive (adversarial) game* is one in which the preferences of each agent are in opposition. A *zero-sum game* is a strictly competitive game in which the agents' payoffs are complementary; *i.e.*, their sum is zero.

- For example:

		B: r	B: s
A	R	$-1, 1$	$0, 0$
	S	$0, 0$	$2, -2$

		B: r	B: s
A	R	-1	0
	S	0	2

- Other examples: chess, poker, football, *etc.*
- Because payoffs complementary, by convention only row player's shown

Dominance-based solutions

Recall that:

Definition (Dominance)

A strategy A is *dominated* by strategy B if for each of the other player's strategies, the outcome of B is at least as preferred as that of the corresponding outcome of A, and for some strategy of the other player it is strictly more preferred.

	a	b	c
A	1	2	4
B	3	2	5

- If A is dominated by B, then B is a *better strategy* regardless of what strategy player 2 plays; *i.e.*, it is a universally *better response*
- Dominated strategies can be disregarded/discarded

Dominance solutions

Exercise

Apply dominance to simplify the following game by eliminating dominated strategies.

	a	b	c
A	1	2	4
B	3	2	5

	a	b	c
A	1	2	4
B	3	2	5

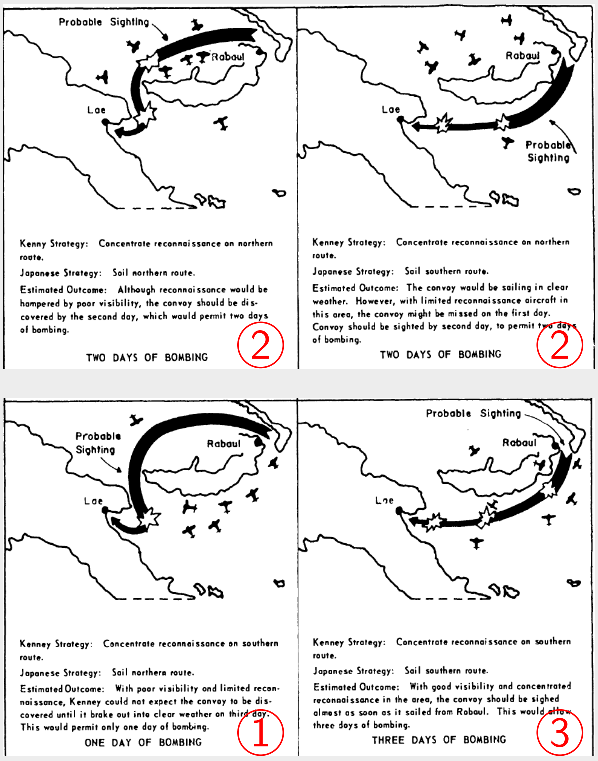
- Dominance helps find solutions by eliminating strategies that neither player will play
- The plays left after dominance in the game above are (B,a) and (B,b)—are these satisfactory solutions?

Battle: table representation

n s

N

S



Convoy

Allies		n	s
	N	2	2
	S	1	3

- Rows = Allies' general's (A) actions/strategies
  Columns = Convoy's general's (C) actions
- Abstraction: row player and column player
- Payoffs: number of bombing days

The battle of the Bismarck Sea

- The battle of the Bismarck Sea is a zero-sum game with imperfect information (neither the convoy Captain nor Allies' General know the other's move*)
- Payoffs are assumed to be complementary

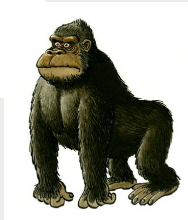
C

A		n	s
	N	2	2
	S	1	3

- Accordingly, the column player prefers outcomes with *smaller* values in the table
- The Battle of the Bismarck Sea is *iterated dominance solvable*

Meet Alice and Bob

Bob



Alice

Strictly competitive, non zero-sum games

- The coconut game is a competitive game that is not zero-sum:

		B			
		W/w C/w	W/w C/c	W/c C/w	W/c C/c
A	W	$0, 0$	$0, 0$	$9, 1$	$9, 1$
	C	$4, 4$	$5, 3$	$4, 4$	$5, 3$

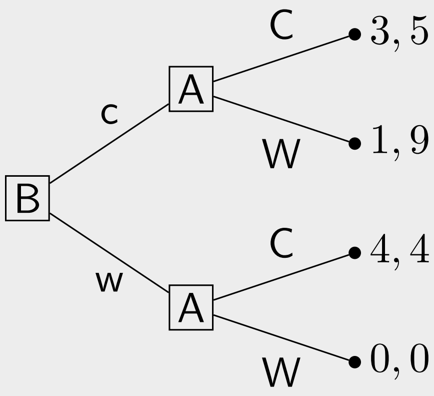
	W/c C/w
W	$9, 1$
C	$4, 4$

	W/c C/w
W	$9, 1$

- Dominance implies that the players should choose strategies: Alice: Wait, Bob: opposite of Alice (*i.e.*, climb if Alice waits, and wait if Alice climbs); compare with *Maximin* (C) which has a value of 4

Reversing roles

- What if Bob moves first?



		A			
		w/W c/W	w/W c/C	w/C c/W	w/C c/C
B	w	0, 0	0, 0	4, 4	4, 4
	c	1, 9	3, 5	1, 9	3, 5

	w/C c/W
w	4, 4
c	1, 9

	w/C c/W
w	4, 4

- Bob, by moving first, causes Alice to climb!

Summary

- Behaviour of other rational agents makes multi-agent decisions more complex:
  - information about other agents' preferences
  - assumption of rationality
- Games can be represented in normal (table/matrix) and extensive (tree) forms
- Zero-sum (constant sum) games: *e.g.*, Bismarck Sea battle
- Strictly competitive non zero-sum games: *e.g.*, Coconut game
- Used multi-lateral (iterated) dominance to narrow-in on admissible solutions